Grade 8 Mathematics, Quarter 4, Unit 4.1

Applying Systems of Linear Equations

Overview

Number of instructional days: 5 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Solve math problems leading to two linear equations in two variables. (For example, given coordinates for 2 pairs of points, determine whether the lines will intersect.) (3 days)
- Connect real-world problem scenarios to a linear mathematical representation. (embedded throughout)
- Interpret the solution to a system of linear equations in the context of a problem situation. (embedded throughout)
- Use an efficient method (graphing, substitution, elimination) to solve a system of linear equations formed from a problem scenario. (1 day)

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem and restate it in their own words.
- Analyze given information to develop possible strategies for solving the problem.
- Identify and execute appropriate strategies to solve the problem.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas.
- Analyze mathematical relationships to draw conclusions.

Attend to precision.

- Communicate their understanding of math to others.
- Specify units of measure, labels, etc.

Essential questions

- How can you determine when a problem scenario (real-world or math problem) can be modeled by a system of linear equations?
- How can you determine which method for solving a system of linear equations is best to use given a problem scenario?
- What is the meaning of the solution of a system of linear equations in the context of a problem scenario?
- What are examples of real-world situations that can be modeled by a system of linear equations?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations	**8.EE**

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students began demonstrating understanding of equality in grades K–3 by finding what makes an open sentence true through adding, subtracting, multiplying or dividing. In grades 6 and 7, students showed equivalence between two expressions by using models of different representations of the expressions and solving one and two-step equations with rational numbers. In grade 8, unit 3.1, students learned to solve multistep equations with rational coefficients, the distributive property, and variables on both sides of the equals. In units 3.2 and 3.3, students learned about functions, how to represent them in various ways, and derived the formula for slope-intercept form for an equation of a line. In unit 4.1, students learn how to solve a system of linear equations graphically and algebraically that have one solution, no solutions or infinite solutions. Students also learn to solve systems of linear equations through inspection. Students will be able to connect that graphical solution to a system of equations matches to the algebraic solutions for the same system of linear equations.

Current Learning

This unit is a continuation of the content from unit 4.1, focusing on solving word problems. Students solve real-world and math problems leading to two linear equations in two variables. Students can determine if two lines intersect, given two pairs of points.

Future Learning

In the high school, students will demonstrate understanding of equality by solving equations, systems of equations or inequalities and interpreting the solutions algebraically and graphically; by factoring, completing the square, using the quadratic formula, and graphing quadratic functions to solve quadratic equations; and by analyzing the effect of simplifying radical or rational expressions on the solution set of equations involving such expressions.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics,* "Stasis and change presents a conceptually rich theme across the grades K–3 curriculum. It has the potential to tie together patterns, functions, and algebra." (pp. 136–149)

Grade 8 Mathematics, Quarter 4, Unit 4.2

Scatterplots and Line of Best Fit

Overview

Number of instructional days: 8 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Construct scatter plots for bivariate measurement data to investigate patterns of association between two quantities. (1 day)
- Interpret scatter plots by describing patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (1 day)
- Informally construct a line of best fit and determine the accuracy by judging the closeness of the data points to the line. (1 day)
- Calculate the equation of the linear model to solve problems involving two sets of data (informal line of best fit). (1 day)
- Give a verbal explanation of the line of best fit in the context of the problem situation (interpret slope and intercept). (1 day)
- Construct and interpret a two-way table summarizing data on two variables. Use relative frequencies to describe possible associations between the two variables. (For example, compare students with or without a curfew on school nights and whether or not they have chores at home.) (2 days)

Mathematical practices to be integrated

Model with mathematics.

- Apply the math they know to solve problems arising in everyday life, society and the workplace.
- Comfortable making assumptions and approximations.
- Identify important quantities in a practical situation and map their relationships using such tools as graphs.
- Analyze mathematical relationships to draw conclusions.

Attend to precision.

- Specify units/labels.
- Communicate their understanding of mathematics to others.

Essential questions

- How can graphing ordered pairs determine whether two sets of numerical data are related?
- How do you informally construct a line of best fit and calculate the equation of its linear model?
- Explain how to construct and interpret a two-way table to describe possible associations between the two variables.
- Why is a scatterplot a good way to show and describe the relationship between two sets of data?
- How do outliers affect the line of best fit and the generalizations you make from that line?
- By interpreting the slope and intercept, what can you learn about the data from your line of best fit?

Written Curriculum

Common Core State Standards for Mathematical Content

Statistics and Probability	8.SP

Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grades K–3, students worked with data in context and used the data to determine more, less or equal. In grades 4 and 5, students learned to represent and interpret data. Grade 6 students developed an understanding of statistical variability, and summarized and described distributions. Grade 7 students used random sampling to draw inferences about a population. They also learned to draw informal comparative inferences about two populations, and investigated chance processes and evaluated probability models.

Current Learning

In this unit, students constructed and interpreted scatterplots for bivariate measurement data to investigate patterns of association between two quantities. Students learn that straight lines are widely used to model relationships between two quantitative variables. They informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line. Students also use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. Students will also understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

Future Learning

Students will continue to analyze patterns, trends, or distributions of data in a variety of contexts by calculating and analyzing measures of dispersion (standard deviation, variance, and percentiles). Students will interpret the data through use of the correlations coefficient (r) and the coefficient of determination (r^2).

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics,* “Stasis and change presents a conceptually rich theme across the grades K–3 curriculum. It has the potential to tie together patterns, functions, and algebra” (pp. 136–149).

Grade 8 Mathematics, Quarter 4, Unit 4.3

Transversals and Angle Relationships

Overview

Number of instructional days: 10 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Use informal arguments to prove that the sum of the angles in a triangle is 180°. (2 days)
- Use informal arguments to prove that an exterior angle of a triangle is the sum of the two remote interior angles. (2 days)
- Use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal. (3 days)
- Use informal arguments to establish facts about the similarity of two triangles through angle-angle similarity. (2 days)

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Habitually ask *why* and seek an answer to that question.
- Understand and use prior learning in constructing arguments.
- Compare the effectiveness of two plausible arguments.

Attend to precision.

- Specify units of measure and label parts of graphs.
- Communicate understanding of mathematics to others.

Essential questions

- How can you show that the angles in a triangle add up to 180°?
- Why is the exterior angle of a triangle equal to the two remote interior angles?
- What angle relationships are formed when parallel lines are cut by a transversal?
- How do you show that triangles are similar by using their angle measurements?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Beginning in grade 4, students recognized angles as geometric shapes that are formed whenever two rays share a common endpoint, and they understood concepts of angle measurements. In grade 7, students learned to use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.

Current Learning

In grade 8, students are using informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. These concepts are introduced, reinforced, and mastered at this level.

Future Learning

In high school geometry, students will continue learning of congruence as it is related to triangles. Furthermore, they will begin proving theorems about lines, angles, triangles, and parallelograms.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics states, "To understand angles, students must understand the various aspects of the angle concept. They must overcome difficulties with orientation, and discriminate angles as critical parts of geometric figures."

"Related topics include parallel and perpendicular lines. Both difficult concepts for students in some applications." (p. 164)

Curriculum Focal Points states, "Students prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lies. Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances. They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles." (p. 39)

Grade 8 Mathematics, Quarter 4, Unit 4.4
Volume

Overview

Number of instructional days: 6 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Know the formulas for the volume of cylinders, cones, and spheres. (Use cube root symbols to represent solutions to equations of the form $x^3 = p$, where p is a positive rational number.) (2 days)
- Solve problems involving volume of cylinders, cones, and spheres. (Express all measures using appropriate units.) (3 days)

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain to themselves the meaning of the problem.
- Plan a solution pathway.

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Identify important quantities in a practical situation using such tools as diagrams and formulas.

Use appropriate tools strategically.

- Use tools when solving a mathematical problem and to deepen their understanding of concepts (i.e., graphing calculators, graph paper, measurement devices).

Attend to precision.

- Specify units of measure.
- Strive for accuracy.

Look for and make use of structure.

- Apply and discuss properties.

Essential questions

- How do you calculate the volume of cylinders, cones, and spheres?

Where would you use the formulas to find volume in the real world?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	8.G

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Expressions and Equations	8.EE

Work with radicals and integer exponents.

8.EE.2 Use ~~square root and~~ cube root symbols to represent solutions to equations of the form ~~$x^2 = p$ and~~ $x^3 = p$, where p is a positive rational number. ~~Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.~~

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 5, students learned to understand the concepts of volume, including relating volume to multiplication and addition. In grade 6, students found the volume of a right rectangular prism with fraction edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and showing that the volume is the same as would be found by multiplying the edge lengths for the prism.

They applied the formulas to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical concepts. In grade 7, students worked with angle measures, area, surface area, and volume of triangles, quadrilaterals, polygons, cubes, and right prisms.

They have acquired a well-developed set of geometric measurement skills. In grade 8 units 2.1 and 2.2, students learned and practiced the concept of finding cubes and cube roots of small perfect cubes. They also learned and practiced their ability to approximate the rational equivalent of non-perfect cubes.

Current Learning

In this unit, students demonstrate knowledge of the formulas to find the volume of cylinders, cones, and spheres. They use them to solve real-world and mathematical problems.

Future Learning

In high school geometry, students will give an informal argument to justify the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. They will also give an informal argument, using Cavalieri's Principle, to justify the formulas for the volume of a sphere and other solid figures. Furthermore, they will use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Additional Findings

According to *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics, "*By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the volumes of prisms and cylinders. They select appropriate two- and three-dimensional shapes to model real-world situations and solve a variety of problems involving surface areas, areas, circumferences of circles, and volumes of prisms and cylinders" (p. 37).