## Grade 8 Mathematics, Quarter 3, Unit 3.1

# Define, Evaluate, and Compare Functions

## Overview

**Number of instructional days:** 15 1 day assessment (1 day = 45–60 minutes)

### Content to be learned

- Understand that a function is a rule that assigns each input to exactly one output. (2 days)
- Understand that the graph of a function is a set of ordered pairs. (1 day)
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). ***see CCSS 8F.2** (2 days)*
- Graph linear relationships and interpret the unit rate as the slope. (1 day)
- Formally calculate the slope of a line. (2 days)
- Understand that the equation $y = mx + b$ defines a linear function whose graph is a straight line. (2 days)
- Give examples of equations that are not linear represented in various ways (graphically, table, equation, verbally). (2 days)
- Use similar triangles to demonstrate that the slope is the same between any two points on a line. (2 days)

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.

Model with mathematics.

- Analyze mathematical relationships to draw conclusions.
- Identify important quantities in a practical situation and map their relationships using diagrams, two-way tables, graphs, flowcharts, and formulas.

Look for and express regularity in repeated reasoning.

- Look for mathematically sound shortcuts.
- Use repeated applications to generalize properties.

### Essential questions

- How can you determine from a graph, table, and equation if a relationship is linear or nonlinear?
- What does slope mean? How would you determine the meaning of slope in the context of a problem situation?
- What is a function?
- What are the different ways that you can calculate slope?
- How does an input-output table relate to a graph?
- What can you compare and interpret from two functions that are represented in different ways?
- Explain how the ratios of the sides of similar triangles formed by the points on a line relate to the slope.

# Written Curriculum

## Common Core State Standards for Mathematical Content

| **Functions** | **8.F** |
|---|---|

**Define, evaluate, and compare functions.**

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

[1] Function notation is not required in Grade 8.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

| **Expressions and Equations** | **8.EE** |
|---|---|

**Understand the connections between proportional relationships, lines, and linear equations.**

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

8.EE.6 Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; ~~derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$~~.

## Common Core Standards for Mathematical Practice

**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Clarifying the Standards

*Prior Learning*

In grades 4 and 5, students analyzed patterns in relationships. Students formed ordered pairs consisting of corresponding terms from two patterns and graphed the ordered pairs on the coordinate plane. In grade 6, students solved unit rate problems including those involving unit pricing and constant speed. In grade 7, students decided whether two quantities were in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. Students identified the unit rate in tables, graphs, equations, diagrams, and verbal descriptions.

*Current Learning*

In grade 8, students learn that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and corresponding output. Students also learn to compare the properties of two functions when they are represented in different ways. They learn the slop-intercept form of an equation and how to calculate slope in a variety of ways. Students also learn the difference between linear and nonlinear functions and can determine if a function is linear or nonlinear in this grade.

*Future Learning*

In high school, students will continue to learn about functions. They will understand the concpet of a function and use function notation, interpret functions that arise in applications in terms of the context, and analyze functions using different representations. Students will then build a function that models a relationship between two quantities and build new functions from existing functions. Students will also construct and compare linear, quadratic, and exponential models and solve problems. They will interpret expressions for functions in terms of the situation they model.

## Additional Research Findings

*A Research Companion to Principles and Standards for School Mathematics* indicates that graphs, diagrams, charts, number sentences, formulas, and other representations play an increasingly important role in mathematical activities (pp. 250–261).

## Grade 8 Mathematics, Quarter 3, Unit 3.2

# Use Functions to Model Relationships

## Overview

**Number of instructional days:** 15 1 day assessment (1 day = 45–60 minutes)

### Content to be learned

- Identify the slope and $y$-intercept from a verbal description, two ordered pairs, a table, or a graph. (4 days)
- Derive the slope-intercept form for the equation of a line given a verbal description, two ordered pairs, a table, or a graph. (6 days)
- Describe the relationship between two quantities by analyzing a graph (i.e., where it is increasing, decreasing, linear, or nonlinear; for example, distance vs. time and story graphs). (2 days)
- Create a graph from a verbal description of a function (when it is increasing, decreasing, linear, nonlinear; for example, distance vs. time and story graphs). (2 days)

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.

Model with mathematics.

- Apply known mathematics to solve problems arising in everyday life, society, and the workplace.
- Identify important quantities in a practical situation and map their relationship using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas.
- Analyze mathematical relationships to draw conclusions.

Attend to precision.

- Specify units/labels (e.g., graphs, axes, titles; all representations).

Look for and express regularity in repeated reasoning.

- Look for mathematically sound shortcuts.
- Use repeated applications to generalize properties.

### Essential questions

- Given a graph, table, set of ordered pairs, or a verbal description, how do you derive the slope-intercept form of an equation?
- Given a verbal description, how can you create the graph of a function?
- How can you determine the slope and $y$-intercept from a graph, table, equation, ordered pairs, or a verbal description?
- What is the process you would use to create an accurate description (story) of a graph of a function?

# Written Curriculum

## Common Core State Standards for Mathematical Content

| Functions | 8.F |
|---|---|

**Use functions to model relationships between quantities.**

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ($x$, $y$) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

| Expressions and Equations | 8.EE |
|---|---|

**Understand the connections between proportional relationships, lines, and linear equations.**

8.EE.6 ~~Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane;~~ derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.

## Common Core Standards for Mathematical Practice

**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Clarifying the Standards

*Prior Learning*

In grades 4 and 5, students analyzed patterns in relationships. They formed ordered pairs consisting of corresponding terms from two patterns and graphed the ordered pairs on the coordinate plane. Grade 6 students solved unit rate problems including those involving unit pricing and constant speed. In grade 7, students decided whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. Students identified the unit rate in tables, graphs, equations, diagrams, and verbal descriptions. In grade 8, unit 3.2, students learned that a function is a rule that assigns to each input exactly one output.

Students also learned how to calculate slope and to identify the difference between linear and nonlinear functions.

*Current Learning*

In unit 3.3, students learn to identify slope and *y*-intercept in graphs, tables, equations, ordered pairs, and a verbal description. Furthermore, students learn to derive the slope-intercept form of a line from graphs, tables, equations, ordered pairs, and verbal descriptions. Students also learn how to describe the changes in the graph of a function (i.e., increasing, decreasing, linear or nonlinear) and can create a graph of a function given the verbal description.

*Future Learning*

In high school, students will continue to learn about functions. They will understand the concept of a function and use function notation, interpret functions that arise in applications in terms of the context, and analyze functions using different representations. Students will then build a function that models a relationship between two quantities and will build new functions from existing functions. Students will also construct and compare linear, quadratic, and exponential models and solve problems. They will interpret expressions for functions in terms of the situation they model.

## Additional Research Findings

*A Research Companion to Principles and Standards for School Mathematics* indicates that graphs, diagrams, charts, number sentences, formulas, and other representations play an increasingly important role in mathematical activities (pp. 250–261).

## Grade 8 Mathematics, Quarter 3, Unit 3.3

# Solving Systems of Linear Equations

## Overview

**Number of instructional days:** 17 1 day assessment (1 day = 45–60 minutes)

### Content to be learned

- Solve systems of linear equations algebraically (substitution, elimination) and by estimation (graphing) that have one solution, no solution (parallel lines), and infinite solutions (same line). (13 days)
- Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs. (2 days)
- Solve systems of simple linear equations by inspection (i.e., know that $3x + 2y = 5$ and $3x+ 2y = 6$ have no solution because they cannot simultaneously equal 5 and 6). (1 day)

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze given information to develop possible strategies for solving the problem.
- Identify and execute appropriate strategies to solve the problem.
- Evaluate progress toward the solution and make revisions if necessary.
- Check their answers using a different method, and continually ask, "Does this make sense?"

Reason abstractly and quantitatively.

- Use varied representations and approaches when solving problems.
- Make sense of quantities and their relationships in problem situations.

Model with mathematics.

- Apply the mathematics they know t solve problems arising in everyday life, society and the workplace.
- Analyze mathematical relationships to draw conclusions.
- Identify important quantities in a practical situation and map their relationships using such tools as graphs, formulas, etc.

Attend to precision.

- Communicate their understanding of mathematics to others.
- Strive for accuracy.

### Essential questions

- What is the meaning of the solution to a system of linear equations?
- How does a graphical solution to a system of linear equations relate to an algebraic solution?
- By observation, how can you determine if a system of linear equations has one solution, no solutions, or infinite solutions?
- What are the different methods for solving a system of two linear equations?

## Written Curriculum

### Common Core State Standards

| **Expressions and Equations** | **8.EE** |
|---|---|

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

### Standards for Mathematical Practices

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Clarifying the Standards

*Prior Learning*

Students began demonstrating understanding of equailty in grades K–3 by finding what makes an open sentence true through adding, subtracting, multiplying or dividing. In grades 6 and 7, students showed equivalence between two expressions by using models of different reprentations of the expressions and solving one- and two-step equations with rational numbers. In grade 8, unit 3.1, students learned to solve multistep equations with rational coefficients, the distributive property, and variables on both sides of the equals. In units 3.2 and 3.3, students learned about functions and how to represent them in various ways; they also derived the formula for slope-intercept form for an equation of a line.

*Current Learning*

In grade 8, students learn how to graphically and algebraically solve a system of linear equations that have one solution, no solutions or infinite solutions. Students also learn to solve systems of linear equations through inspection. Students will be able to connect that graphical solution to a system of equations matches to the algebraic solutions for the same system of linear equations.

*Future Learning*

In high school, students will demonstrate understanding of equality by solving equations, systems of equations, or inequalities and by interpreting the solutions algebraically and graphically; by factoring, completing the square, using the quadratic formula, and graphing quadratic functions to solve quadratic equations; and by analyzing the effect of simplifying radical or rational expressions on the solution set of equations involing such expressions.

## Additional Research Findings

According to *A Research Companion to Principles and Standards for School Mathematics,* "Stasis and change presents a conceptually rich theme across the grades K–3 curriculum. It has the potential to tie together patterns, functions, and algebra" (pp. 136–149).