

# Grade 8 Mathematics, Quarter 2, Unit 2.1

## Rational and Irrational Numbers

### Overview

**Number of instructional days:** 10 1 day assessment (1 day = 45–60 minutes)

#### Content to be learned

- List the characteristics of rational and irrational numbers. (1 day)
- Evaluate square roots and cube roots (small perfect squares and cube roots of small perfect cubes). (1 day)
- Compute whole numbers or fractional bases with whole number exponents. (1 day)
- Compare and order all rational numbers (integers, scientific notation, absolute value, whole number and fractional bases with whole number exponents, benchmark percents) and all irrational numbers ( $\sqrt{2}$ ,  $\pi$ ) using number lines or through estimation. (3 days)
- Convert decimals to fractions, including repeating decimals. (1 day)
- Express numbers in scientific notation. (1 day)

#### Essential questions

- How are rational and irrational numbers similar and different?
- What strategies would you use to mentally compute irrational numbers?
- How would you convert a repeating decimal into a rational number?
- How would you order and compare rational and irrational numbers?

#### Mathematical practices to be integrated

Attend to precision.

- Communicate using proper vocabulary and clear definitions pertaining to types of numbers (real, rational, etc.).
- State the meaning of the equality/inequality symbols.
- Calculate accurately and efficiently.
- Express numerical answers with precision when comparing numbers.

Construct viable arguments and critique the reasoning of others.

- Categorize numbers appropriately and justify reasoning.
- Order numbers appropriately and justify reasoning.
- Analyze situations by breaking them into cases such as rational and irrational (i.e., justify why all natural numbers are whole numbers).

- What is the relationship between an exponent and its base in a problem?
- What does it mean to find the square or cube root of a number?
- How do you express very large quantities or very small quantities in scientific notation?

## Written Curriculum

### Common Core State Standards for Mathematical Content

#### The Number System

**8.NS**

#### Know that there are numbers that are not rational, and approximate them by rational numbers.

- 8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
- 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). *For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

#### Expressions and Equations

**8.EE**

#### Work with radicals and integer exponents.

- 8.EE.2 ~~Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number.~~ Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.
- 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.*

### Common Core Standards for Mathematical Practice

#### 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

### Clarifying the Standards

#### *Prior Learning*

In earlier grades, students developed an understanding of various rational numbers. From kindergarten to grade 2, they used place value and properties of addition and subtraction for numbers up to 20. Students also worked on understanding place value and the properties of operations for addition and subtraction. Grade 3 students continued work with place value, and students began to understand fractions. In grade 4, students continued to build their understanding of fractions and began using decimals. They also began working with changing fractions to decimals, comparing fractions and decimals, and determining which is larger by using inequality symbols.

In grade 5, students used decimal fractions to the thousandth place and continued to compare them using inequality symbols. They also learned to multiply and divide fractions. In grade 6, students were introduced to absolute value, percents, and number lines to position two numbers. They wrote and interpreted statements for order of rational numbers. By the end of grade 6, students had extended their previous understanding of number and the ordering of numbers to the full system of rational numbers, which included negative rational numbers and in particular negative integers. They reasoned about the order and absolute value of rational numbers and about the location of the points in the four quadrants of the coordinate plane.

Grade 7 students developed an understanding of number, recognizing fractions, decimals, and percents as different representations of rational numbers. They extended addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, multiplication and division. Students used the arithmetic of rational numbers as they formulated expressions and equations in one variable and used these equations to solve problems.

#### *Current Learning*

Students extend the learning of relative magnitude of numbers by ordering and comparing rational and irrational numbers. They evaluate square roots or small perfect squares and cube roots of small perfect cubes and express numbers in scientific notation. Students use equality and inequality symbols and the number line to order and compare the rational and irrational numbers. They learn that numbers that are not rational are called *irrational*. Students understand informally that every number has a decimal expansion, and they convert a decimal expansion that repeats eventually into a rational number. Students also learn to estimate the decimal approximation of square roots that are not perfect. These concepts are introduced and reinforced, and some are mastered at this level.

*Future Learning*

Students will continue to work with the relative magnitude of various rational and irrational numbers in algebra 1, and they will continue to refine and use their mental computation strategies in problem situations.

**Additional Findings**

According to *Principles and Standards for School Mathematics*, students should acquire the ability to make reasonable estimates and use benchmarks to order and compare numbers in sixth through eighth grades. Students should be able to think flexibly about the sizes of rational and irrational numbers and should be given the opportunity to share different estimation methods for solving various problems (pp. 214–221).

Additionally, *A Research Companion to Principles and Standards for School Mathematics* states that students should be given the opportunity to represent all numbers in general and abstract ways based on their prior learning. The goal of problem solving is for students to generalize their learning applicable to various situations (pp. 263–273).

Grade 8 Mathematics, Quarter 2, Unit 2.2

# Radicals and Operations / Properties of Integer Exponents

## Overview

**Number of instructional days:** 12 1 day assessment (1 day = 45–60 minutes)

### Content to be learned

- Apply the properties of integer exponents to generate equivalent numerical expressions. (For example,  $3^2 \cdot 3^5 = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ .) (5 days)
- Perform operations with numbers expressed in scientific notation, including problems where both decimals and scientific notation are used. (5 days)
- Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. (1 day)
- Interpret scientific notation that has been generated by technology.

### Essential questions

- How do you use the rules of integer exponents to simplify expressions?
- How do you multiply numbers in scientific notation?
- How do you divide numbers in scientific notation?

### Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Consider units involved.
- Know and flexibly use different properties of operations and objects.

Attend to precision.

- Specify units/labels.
- Clarify meaning of symbols.
- Use appropriate degree of precision.

Look for and make use of structure.

- Apply and discuss properties.

- How do you add and subtract numbers in scientific notation?
- When is it appropriate to use scientific notation?
- How do you interpret a number from the calculator that is given in scientific notation?

## Written Curriculum

### Common Core State Standards for Mathematical Content

#### Expressions and Equations

**8.EE**

#### Work with radicals and integer exponents.

- 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .
- 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### Common Core Standards for Mathematical Practice

#### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

#### 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the

significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

### Clarifying the Standards

#### *Prior Learning*

In grade 5, students learned how to explain patterns in the number of zeros of the product when multiplying a number by powers of 10. Students learned to use whole-number exponents to denote powers of 10. In grade 6, students continued learning expressions and learned to write and evaluate numerical expressions involving whole-number exponents. Also, students performed arithmetic operations, including those involving whole-number exponents. In unit 2.1, grade 8 students worked with rational and irrational numbers. Students learned to express numbers in scientific notation, calculate perfect squares and perfect cubes, and estimate irrational numbers.

#### *Current Learning*

In this unit, students are learning the rules for integer exponents, how to perform operations with numbers expressed in scientific notation, and interpret scientific notation generated by technology. These concepts are introduced and reinforced at this level.

#### *Future Learning*

In high school, students will continue to work with properties of exponents, expanding their study from integer to rational exponents. Specifically, students will be explaining how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. Students will also learn to rewrite expressions involving radicals and rational exponents using the properties of exponents.

### Additional Findings

*Principles and Standards for School Mathematics* states, “Student should develop a deeper understanding of very large and very small numbers and of various representations of them. They should also be able to judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities. They should also judge the reasonableness of numerical computations and their results.” “They need to become familiar with different ways of representing numbers. As part of their developing technological facility, students should become adept at interpreting numerical answers on calculator or computer displays” (pp. 290–291).



Grade 8 Mathematics, Quarter 2, Unit 2.3  
**Solving Linear Equations in One Variable**

**Overview**

**Number of instructional days:** 20 1 day assessment (1 day = 45–60 minutes)

**Content to be learned**

- Solve multistep linear equations with rational coefficients, variables on both sides, distributive property, and combining like terms. (12 days)
- Translate a problem situation into an algebraic equation and solve for the unknown. (2 days)
- Solve equations that have one solution, many solutions, and no solutions. (5 days)

**Mathematical practices to be integrated**

Model with mathematics.

- Solve problems using equations.
- Interpret results in the context of the situation.
- Reflect on the reasonableness of the answer.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
- See complicated things, such as algebraic expressions, as single objects or as being composed of several objects.
- Step back for an overview and shift perspective.

Attend to precision.

- Specify units of measure and label parts of graphs and charts.
- Strive for accuracy.

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated, and look both for general methods and shortcuts.
- Evaluate the reasonableness of intermediate results.

**Essential questions**

- How do you maintain equivalence when solving equations?
- What processes would you use when solving a multistep equation?
- Can equations that appear to be different be equivalent?
- How do you develop an equation to model a problem situation?
- How many possible solutions are there to a linear equation in one variable? Describe each one.

## Written Curriculum

### Common Core State Standards for Mathematical Content

<b>Expressions and Equations</b>	<b>8.EE</b>
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#### Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

- a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers).
- b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

### Common Core Standards for Mathematical Practice

#### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### 6 Attend to precision.

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**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Clarifying the Standards***Prior Learning*

In grade 4, students began to represent and solve multistep word problems using equations with a letter standing for the unknown quantity. Grade 6 students learned to reason about and solve one-variable equations. Specifically, students understood solving an equation or inequality as a process of answering a question. They used substitution to determine whether a given number in a specified set makes an equation true. Students also used variables to represent numbers and write expressions when solving a real-world or mathematical problem and understood that a variable can represent an unknown number.

Students also solved real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  for cases in which  $p$ ,  $q$ , and  $x$  are all nonnegative rational numbers. Students also learned the distributive property and how to combine like terms as it relates to expressions. In grade 7, students used variables to represent quantities in real-world or mathematical problems, and constructed simple equations and inequalities to solve problems by reasoning about the quantities. They solved word problems leading to equations of the form  $px + q = r$  and  $P(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. They solved equations of these forms fluently. They also compared an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

*Current Learning*

Students demonstrate understanding of equality by showing equivalence between two expressions using models or different representations of the expression (e.g., algebra tiles). They solve multistep linear equations with rational coefficients and equations with variables on both sides. They show that two expressions are or are not equivalent by applying the commutative, associative, or distributive properties; order of operations; or substitution. Students also solve equations that have no solutions, rational solutions, or infinitely many solutions. This concept is practiced and reinforced at this level.

*Future Learning*

In the high school, students will continue to solve equations and translate problem situations into equations. Students will explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Students also solve linear equations in one variable, including equations with coefficients represented by letters. Students will rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

**Additional Research Findings**

According to *Principles and Standards for School Mathematics*, “in the middle grades, students should also learn to recognize and generate equivalent expressions, solve linear equations, and use simple formulas. Whenever possible, the teaching and learning of algebra can and should be integrated with other topics in the curriculum” (p. 223).