Grade 8 Mathematics, Quarter 1, Unit 1.1

Transformations

Overview

Number of instructional days: 10 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Solve problems on and off the coordinate plane involving reflections, using line segments, angles, and parallel lines reflected over the *y*-axis, *x*-axis, and the line $y = x$. * (3 days)
- Solve problems on and off the coordinate plane involving translations, using line segments, angles, and parallel lines.* (3 days)
- Solve problems on and off the coordinate plane involving rotations, using line segments, angles, and parallel lines.* (3 days)
- Use available technology to explore transformations.

*Two-dimensional shapes are not covered in this unit.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Habitually ask *why* and seek an answer to that question.
- Understand and use prior learning in constructing arguments.
- Justify conclusions, communicate them to others, and respond to the arguments of others.

Attend to precision.

- Strive for accuracy.
- Specify units of measure and label parts of graphs.
- Communicate understanding of mathematics to others.

Essential questions

- What are the similarities and differences among translations, rotations, and reflections?
- What is the process used to rotate a line segment, an angle, or parallel lines?
- What is the process used to reflect a line segment, an angle, or parallel lines?
- What is the process used to translate a line segment, an angle, or parallel lines?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 4, students began to draw points, lines, line segments, rays, angles, and perpendicular and parallel lines. They learned to identify them in two-dimensional figures. Students also learned to recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. They identified line-symmetric figures and drew lines of symmetry. In grade 5, students learned to graph points on the coordinate plane to solve real-world and mathematical problems. In grade 6, students continued to use their knowledge of graphing points on a coordinate plane to draw polygons in the coordinate plane given the vertices. Furthermore, students find the length of the sides of the polygons using the coordinates of the vertices.

Current Learning

Students verify experimentally the properties of rotations, reflections, and translations of line segments, angles, and parallel lines. These concepts are at the developmental, reinforcement, and drill-and-practice level.

Future Learning

In grade 8, unit 1.2, students will use their knowledge of translations, reflections, and rotations with two-dimensional shapes to determine congruence. In unit 1.3, students will use their knowledge of dilations, translations, rotations, and reflections to determine similarity with two-dimensional shapes. In geometry, students will connect transformations to functions and describe the translations, rotations, and reflections that take one figure to another. They will also draw a new transformed figure using various math tools (i.e., graph paper, tracing paper, geometry software).

Additional Findings

Curriculum Focal Points states that students should "describe sizes, positions, and orientations of shapes under information transformations such as flips, turns, slides and scaling." (p. 37)

Principles and Standards for School Mathematics states, "Young children come to school with intuitions about how shapes can be moved. Students can explore motions such as slides, flips, and turns by using mirrors, paper folding, and tracing. Later, their knowledge about transformations should become more formal and systematic. In grades 3–5, students can investigate the effects of transformations and begin to describe them in mathematical terms." (p. 43)

Grade 8 Mathematics, Quarter 1, Unit 1.2

Congruence

Overview

Number of instructional days: 10 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Apply translations, rotations, and reflections to two-dimensional figures using coordinates. (4 days)
- Understand congruence of two-dimensional figures through a sequence of transformations (rotations, reflections, and translations). (3 days)
- Prove that two figures are congruent through a sequence of transformations (rotations, reflections, and translations). (2 days)

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze given information to develop possible strategies for solving a problem.
- Identify and execute appropriate strategies to solve a problem.

Construct viable arguments and critique the reasoning of others.

- Understand and use prior learning in constructing arguments.
- Develop questioning strategies to generate information.
- Justify conclusions, communicate them to others, and respond to the arguments of others.

Essential questions

- What steps would you take to translate, rotate, and reflect a two-dimensional figure on a coordinate grid?
- Using transformations, how can you prove that two 2-dimensional figures are congruent?
- Given two congruent figures, how do you determine the sequence of translations, reflections, and rotations necessary to move from one figure to another?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3 Describe the effect of ~~dilations,~~ translations, rotations, and reflections on two-dimensional figures using coordinates.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an

argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In grade 4, students began to draw points, lines, line segments, rays, angles, and perpendicular and parallel lines, and they learned to identify these in two-dimensional figures. They also learned to recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Students identified line-symmetric figures and drew lines of symmetry. In grade 5, students learned to graph points on a coordinate plane by using the x and y coordinates. In grade 6, students learned to draw polygons in a coordinate plane given coordinates for the vertices. In grade 7, students drew, constructed and described geometric figures and described the relationship among them. They solved problems using scale drawings of geometric figures, including reproducing a scale drawing at a different scale. In grade 8, unit 1.1, students learned how to translate, rotate, and reflect angles, line segments, and parallel lines.

Current Learning

Students learn congruence of two-dimensional figures through a sequence of rotations, reflections, and translations. They describe a sequence that exhibits the congruence between two figures. Students also describe the effect of translations, rotations, and reflections on two-dimensional figures using coordinates.

Future Learning

In grade 8, unit 1.3, students will use their knowledge of translations, rotations, and reflections on two-dimensional figures using coordinates to determine similarity about two-dimensional shapes. In geometry, students will connect transformations to functions, describing the translations, rotations, reflections, and dilations that take one figure to another, and they will also draw a new transformed figure using various math tools (i.e., graph paper, tracing paper, geometry software). Students will also use congruence criteria for triangles to solve problems and to prove relationships in geometric figures.

Additional Findings

Principles and Standards for School Mathematics states, "Middle-grades students also need experience in working with congruent and similar shapes. From their earlier work, students should understand that congruent shapes and angles are identical and can be matched by placing one atop the other. Students can investigate congruence and similarity in many settings, including art, architecture, and everyday life." (p. 234)

Grade 8 Mathematics, Quarter 1, Unit 1.3

Similarity

Overview

Number of instructional days: 10 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Describe the effect of dilations on two-dimensional figures using coordinates to prove similarity. (Coordinates are used to find the length of horizontal and vertical lines on the coordinate plane.) (4 days)
- Understand similarity of two-dimensional figures through a sequence of transformations (dilations, rotations, reflections, translations). (3 days)
- Describe the sequence of transformations that shows the similarity between a pair of two-dimensional figures. (2 days)

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Understand and use prior learning in constructing arguments.
- Seek to understand alternative approaches suggested by others and, as a result, adopt better approaches.
- Justify conclusions, communicate them to others, and respond to the arguments of others.

Use appropriate tools strategically.

- Use tools when solving a mathematical problem and to deepen understanding of concepts.
- Make sound decisions about when tools might be helpful, recognizing both the insight to be gained and the limitations.

Essential questions

- Using their coordinates, how can you determine whether or not a pair of two-dimensional figures is similar?
- How do you create a similar figure through a dilation?
- Given two similar figures, what is the sequence of transformations necessary to move from one figure to another?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grade 4, students began to draw points, lines, line segments, rays, angles, and perpendicular and parallel lines, and they learned to identify them in two-dimensional figures. Students also learned to recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. They identified line-symmetric figures and drew lines of symmetry. In grade 5, students learned to graph points on a coordinate plane by using the x and y coordinates.

In grade 6, students learned to draw polygons in a coordinate plane given coordinates for the vertices. In grade 7, students drew, constructed, and described geometric figures, and they described the relationship among them. Students solved problems using scale drawings of geometric figures, including reproducing a scale drawing at a different scale. In grade 8, unit 1.1, students learned transformations of angles, line segments, and parallel lines. In unit 1.2, students learned how to determine congruence of two-dimensional figures through a sequence of translations, rotations, and reflections.

Current Learning

Students learn similarity of two-dimensional figures through a sequence of rotations, reflections, and translations and dilations. They also describe a sequence that exhibits the similarity between two figures. Students also describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Future Learning

In geometry, students will use the definition of *similarity*, in terms of similarity transformations, to decide if two figures are similar. They will also explain transformations. Using transformations, students will explain the meaning of similarity for triangles as equal angles and proportional sides. They will use the properties of similar triangles to establish the angle-angle relationship between two similar triangles. Students will also use similarity of triangles to solve problems and prove relationships. Lastly, they will understand that similarity in right triangles leads to trigonometric definitions.

Additional Findings

Principles and Standards for School Mathematics states, "Investigations into the properties of, and relationships among, similar shapes can afford students many opportunities to develop and evaluate conjectures inductively and deductively. Transformational geometry offers another lens through which to investigate and interpret geometric objects. To help them form images of shape through different transformations, students can use physical objects, figures traced on tissue paper, mirrors or other reflective surfaces, figures drawn on graph paper, and dynamic geometry software. They should explore the characteristics of flips, turns, and slides and should investigate relationships among compositions of transformations. These experiences should help students develop a strong understanding of line and rotational symmetry, scaling, and properties of polygons." (pp. 234–235)

Grade 8 Mathematics, Quarter 1, Unit 1.4

Pythagorean Theorem

Overview

Number of instructional days: 12 1 day assessment (1 day = 45–60 minutes)

Content to be learned

- Apply the Pythagorean Theorem to find the missing side of a right triangle. (Use square roots to represent solutions to equations of the form *x^2 = p*, where *p* is a positive rational number.) (3 days)
- Apply the Pythagorean Theorem to problem-solving situations. (3 days)
- Explain a proof of the Pythagorean Theorem and its converse. (4 days)
- Apply the Pythagorean Theorem to find the distance between two points on a coordinate plane. (1 day)

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Plan a solution pathway.
- Analyze givens, constraints, relationships, and goals.

Model with mathematics.

- Identify important quantities in a practical situation.
- Apply known mathematics to solve problems.
- Analyze relationships mathematically to draw conclusions.

Use appropriate tools strategically.

- Use tools to solve, explore, compare, and visualize problems and to deepen knowledge/understanding.
- Detect errors using estimation and other mathematical knowledge.

Essential questions

- What is the relationship among the three sides of a right triangle?
- How can you use the Pythagorean Theorem to find the distance between two points on a coordinate plane?
- How do you find a missing side of a right triangle?
- How do you use the Pythagorean Theorem in a problem-solving situation?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations 8.EE

Work with radicals and integer exponents.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Geometry 8.G

Understand and apply the Pythagorean Theorem.

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grade 6, students found the area of a right triangle. In grade 7, students solved real-life and mathematical problems involving angle measurements. In grade 8, units 2.1 and 2.2, students learned about squares, cubes, square roots, and cube roots as well as approximating non-perfect squares and cubes.

Current Learning

In this unit, students learn the Pythagorean Theorem and its applications in real-world situations. They find the side length of an unknown side, the distance between two points in a coordinate system, and explain the proof and its converse.

Future Learning

In high school geometry, students will prove various theorems about triangles, including proving the Pythagorean Theorem using triangle similarity. Furthermore, students will learn to use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Additional Findings

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics states, "Students need to explain why the Pythagorean Theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedrals" (p. 20).