## Grade 7 Mathematics, Quarter 4, Unit 4.1

# Populations and Samples

## Overview

**Number of instructional days:** 10 (1 day = 45–60 minutes)

### Content to be learned

- Understand that statistics are a way to gain information about a population through samples.
- Recognize the validity of a sample and whether it appropriately represents the population.
- Understand that random sampling tends to produce representative samples and support valid inferences.
- Use random sampling to draw inferences about a population.
- Generate multiple samples to make predictions or draw conclusions.
- Informally assess two sets of data to identify similarities and differences.
- Use measures of center, mean, and median for numerical data from random samples to draw informal inferences about two populations.
- Use measures of variability, mean absolute deviation, and interquartile range for numerical data from random samples to draw informal inferences about two populations.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them**.**

- Use central tendencies to draw conclusions regarding real-life situations.
- Analyze the relationships between two sets of data.
- Draw conclusions through the analysis of data.
- Plan a solution pathway to appropriately collect and analyze data for a specific situation, rather than jumping into the work.
- Monitor and evaluate progress, changing the sample if it is not representative of the population.
- Transform information into different representations.

Construct viable arguments and critique the reasoning of others.

- Reason inductively about data, making plausible arguments that take into account the context from which the data arose (using central tendencies to draw conclusions).
- Identify flawed data, explaining how it is flawed (i.e., bias, outliers, unrepresentative samples).

Model with mathematics.

- Analyze data to draw conclusions.
- Interpret mathematical results in the context of the situation and reflect on whether the results make sense (i.e., how does the data collected / analyzed connect to the real world? Why would the results be relevant?).

### Essential questions

- Why is it important to represent and analyze data using measures of central tendency?
- What do the measures of center tell you abut the data?
- How can we tell if the sample clearly represents the population we are interested in?
- How can we use multiple samples (or simulation samples) to make estimates or predictions about a population?
- What might be the cause(s) of differences between two (or more) samples from the same population?
- When and where are statistics used in the real world?
- How can visual representations help us describe similarities and differences between two sets of data?

## Written Curriculum

### Common Core State Standards for Mathematical Content

**Statistics and Probability** **7.SP**

**Use random sampling to draw inferences about a population.**

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

**Draw informal comparative inferences about two populations.**

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

## Common Core Standards for Mathematical Practice

### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## Clarifying the Standards

*Prior Learning*

In grades K–3, students learned to generate data and used representations including bar and picture graphs. In grades 4–5, students interpreted differences in data and created line plot representations. Appropriate intervals were displayed on graphic representations also. In grade 6, students developed their ability to think statistically. They recognized various measures of center and saw that these can be used to summarize data.

*Current Learning*

In grade 7, students use random sampling to draw inferences about a population by examining samples that are valid representations of a given population. Students use data from random samples to make inferences about a population and generate multiple samples to make predictions or draw conclusions. Students make comparative inferences to two related sets of data and draw conclusions based on measures of center (mean and median) and measures of variation (mean absolute deviation and interquartile range.)

*Future Learning*

In grade 8, students will continue to collect, display, and analyze data to find patterns, make predictions, and draw conclusions through the use of scatterplots and other bivariate data. In high school, students will study distribution and interpret differences in the shape, center, and spread in the context of data sets. They will understand statistics as a process for making inferences about population parameters based on random samples from a population.

## Additional Findings

According to *Principles and Standards of School Mathematics*, "Prior to the middle grades, students should have had experiences collecting, organizing, and representing sets of data. They should be facile both with representational tools (such as tables, line plots, bar graphs, and line graphs) and with measures of center and spread (such as median, mode, and range). They should have had experience in using some methods of analyzing information and answering questions, typically about a single population. In grades 6–8, teachers should build on this base of experience to help students answer more complex questions, such as those concerning relationships among populations or samples and those about relationships between two variables within one population or sample" (p. 249).

During their time in the middle school, "students should learn to use the mean, and continue to use the median and the mode, to describe the center of a set of data. Although the mean often quickly becomes the method of choice for students when summarizing a data set, their knack for computing the mean does not necessarily correspond to a solid understanding of its mean or purpose (McClain, 1999)" (p. 251).

In this area of collecting and representing data, students "should make observations, inferences, and conjectures, and develop new questions" (pp. 251–252).

Grade 7 Mathematics, Quarter 4, Unit 4.2

# Finding Geometric Measures Algebraically

## Overview

**Number of instructional days:** 10 (1 day = 45–60 minutes)

### Content to be learned

- Write and solve simple equations for an unknown angle in a figure by using facts about supplementary, complementary, vertical and adjacent angles.
- Apply formulas for circumference and area of a circle to solve problems.
- Explain the relationship between the circumference of a circle and its area.

### Mathematical practices to be integrated

Attend to precision.

- "Math language" appropriate vocabulary and precise definitions should be used to describe relationships.
- Clarify meaning of symbols.

Look for and make use of structure.

- Apply and discuss properties.
- Mathematically proficient students look closely to discern a pattern or structure when developing relationships among angles.

### Essential questions

- How are vertical, adjacent, complementary, and supplementary angles related?
- Is it possible for vertical angles to be adjacent angles? Describe a situation to justify your answer.
- How are the circumference and area of a circle related?
- How are the circumference and diameter of a circle related?

# Written Curriculum

## Common Core State Standards for Mathematical Content

**Geometry 7.G**

**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

## Common Core Standards for Mathematical Practice

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## Clarifying the Standards

*Prior Learning*

In grade 3, students recognized area as an attribute of plane figures and understand concepts of area measurement. Students also related area to the operations of multiplication and addition. In grade 4, students began work with points, lines, rays, and angles (right, obtuse, acute), as well as perpendicular and parallel lines. They solved addition and subtraction problems to find unknown angles in a diagram. Students in grade 5 applied angle and line relationships to two-dimensional figures to classify them.

*Current Learning*

In grade 7, students know the formulas for the area ($A = \pi r^2$, where A = area and r = radius) and the circumference ($C = \pi d$ or $C = 2\pi r$, where C = circumference, d = diameter, and r = radius) of a circle and use them to solve problems. Students give an informal derivation of the relationship between the circumference and area of a circle to develop the area formula from the formula for circumference. Grade 7 students use facts about supplementary, complementary, vertical, and adjacent angles in multistep problems to write and to solve simple equations for an unknown angle in a figure.

*Future Learning*

In grade 8, students will use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines cut by a transversal and the angle-angle criterion for similarity of triangles. In high school geometry, students will identify and describe relationships among the radii and chords of circles. They will also need to give informal arguments for the formulas for the circumference and the area of a circle.

## Additional Findings

"Students should come to the study of geometry in the middle grades with informal knowledge about points, lines, planes, and a variety of two- and three-dimensional shapes; with experience in visualizing and drawing lines, angles, triangles, and other polygons; and with intuitive notions about shapes built from years of interacting with objects in their daily lives." (*Principles and Standards for School Mathematics*, p. 233)

## Grade 7 Mathematics, Quarter 4, Unit 4.3

# Geometry Constructions

## Overview

**Number of instructional days:** 5 (1 day = 45–60 minutes)

### Content to be learned

- Draw geometric shapes with given conditions, using ruler and protractor, freehand, and technology, with a focus on triangles.
- Construct triangles from 3 measures of angles or sides and notice when the conditions determine a unique triangle, more than one triangle, or no triangle.

### Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Justify conclusions, communicate them to others, and respond to the arguments of others (ability to determine which conditions form a unique triangle, more than one triangle, or no triangle).
- Understand and use prior learning in constructing arguments.

Use appropriate tools strategically.

- Use tools when solving a mathematical problem and deepen understanding of concepts (e.g., pencil and paper, protractor, ruler, tracing/patty paper, technology).

Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure (ability to determine which conditions form a unique triangle, more than one triangle, or no triangle).
- Apply and discuss properties (conditions that form a unique triangle, more than one triangle, or no triangle).

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated, and look both for general methods and shortcuts (conditions that form a unique triangle, more than one triangle, or no triangle).
- Maintain oversight of the process while attending to details.
- Evaluate the reasonableness of their results.

### Essential questions

- What do you notice about the measures of the sides or the measures of the angles that form a triangle?
- How can triangles be classified?
- What tools can be used to construct a two-dimensional shape and how would they be used?

## Written Curriculum

### Common Core State Standards for Mathematical Content

| Geometry | 7.G |
|---|---|

**Draw, construct, and describe geometrical figures and describe the relationships between them.**

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

### Common Core Standards for Mathematical Practice

**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and

compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Clarifying the Standards

*Prior Learning*

In grade 4, students recognized that angles are formed in geometric shapes. They measured angles in whole number degrees using a protractor and sketched them and recognized right triangles as category and were able to identify them. Grade 5 students were able to describe the attributes of two-dimensional figures and categorize them into a hierarchy. Grade 6 six students began to construct two-dimensional shapes on a coordinate plane to prepare for constructions.

*Current Learning*

Using various tools (freehand, ruler, protractor, technology) students will draw geometric shapes with given conditions with focus on triangles. Given the measures of the 3 sides or angles of a triangle, students determine whether the measures form a unique triangle, more than one triangle, or no triangle.

*Future Learning*

In grade 8, students will start to establish facts about the angle sum and exterior angle of triangles and use models to learn about congruence and similarity. In high school, students will prove theorems about triangles and make formal geometric constructions using a variety of tools and methods.

## Additional Findings

According to *Principles and Standards for School Mathematics,* "Students should come to the study of geometry in the middle grades with informal knowledge about points, lines, planes, and a variety of two and three dimensional shapes; with experience in visualizing and drawing lines, angles, triangles, and other polygons; and with intuitive notions about shapes built from years of interacting with objects in their daily lives.

In middle grades geometry programs, based on these recommendations, students investigate relationships by drawing, measuring, visualizing, comparing, transforming, and classifying geometric objects. Geometry provides a rich context for the development of mathematical reasoning, including inductive and deductive reasoning making and validating conjectures, and classifying and defining geometric objects.

… Middle grades students should explore a variety of geometric shapes and examine their characteristics. Students can conduct these explorations using materials such as geo-boards, dot paper, multiple length cardboard strips with hinges, and dynamic geometry software to create two-dimensional shapes. Students must carefully examine the features of shapes in order to precisely define and describe fundamental shapes, such as special types of quadrilaterals, and to identify relationships among the types of shapes." (p. 233)

*A Research Companion to Principles and standards for School Mathematics* states, "The construct of concept images suggests that although diagrams and pictures can support geometric reasoning, they bring their own set of problems. Research substantiates that when perceiving a diagram for a proof problem, for example, a student must focus on what is essential and dismiss what is inessential – a difficult process for many… Similarly, using manipulatives can facilitate the construction of sound representations of geometric concepts, but they too must be used wisely (Clements & Battista, 1992). Unfortunately, U.S. textbooks only infrequently suggest the use of manipulatives in geometry, and even when they do, the suggested uses are not aimed at developing higher levels of thinking." (p. 155)

### Grade 7 Mathematics, Quarter 4, Unit 4.4

# Two- and Three-Dimensional Figures

## Overview

**Number of instructional days:** 10 (1 day = 45–60 minutes)

### Content to be learned

- Describe the cross-sections that result from slicing three-dimensional figures, namely right rectangular prisms and right rectangular pyramids.
- Solve real-world and mathematical problems involving area of two-dimensional objects composed of quadrilaterals, triangles, and polygons.
- Solve real-world and mathematical problems involving the volume and surface area of three-dimensional objects composed of cubes and right prisms.

### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem and restate it in their own words.
- Check their answers using a different method, and continually ask, "Does this make sense?"

Attend to precision.

- "Math language" appropriate vocabulary; precise definitions.
- Use appropriate degree of precision related the measurement of shapes.

Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure.
- Students can step back for an overview and shift perspective.
- Students can see complicated as either single objects or as being composed of several objects.

### Essential questions

- Without using a formula, how could you determine the surface area of a three-dimensional figure?
- How do you calculate the surface area of three-dimensional figures?
- Without using a formula, how could you determine the volume of a three-dimensional figure?
- How do you calculate the volume of three-dimensional figures?
- How can knowing the shape of the base of a three-dimensional help you name the figure?
- How is finding the volume of a rectangular prism and the volume of a triangular prism alike? Different?
- What relationship(s) do the base and cross-sections of a three-dimensional object have, if any?

# Written Curriculum

## Common Core State Standards for Mathematical Content

**Geometry 7.G**

**Draw, construct, and describe geometrical figures and describe the relationships between them.**

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

## Common Core Standards for Mathematical Practice

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

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## Clarifying the Standards

*Prior Learning*

Work with area began in grade 3. By grades 5 and 6, students moved into volume by finding the volume of right rectangular prisms using unit cubes and the formulas $V = lwh$ and $V = bh$. Grade 6 students used fractional lengths in their volume work. Also in grade 6, students represented three-dimensional figures using nets made up of rectangles and triangles, and used the nets to find surface area of those figures. They applied this work to solving real-world and mathematical problems.

*Current Learning*

In grade 7, students describe the cross-sections from slicing three-dimensional figures, namely right rectangular prisms and right rectangular pyramids. Students also solve real-world and mathematical problems involving the area of two-dimensional figures and the volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

*Future Learning*

In grade 8, students will expand their work with volume and will be able to find the volumes of cones, cylinders, and spheres, and will use them to solve real-world and mathematical problems. Students will continue their work in high school geometry through explaining volume formulas and using them to solve problems.

## Additional Findings

*Principles and Standards for School Mathematics* states:

"Students' skills in visualizing and reasoning about spatial relationships are fundamental in geometry. Some students may have difficulty finding the surface area of three-dimensional shapes using two-dimensional representations because they cannot visualize the unseen faces of the shapes. Experience with models of three-dimensional shapes and two-dimensional 'nets' is useful in such visualization. Students also need to examine, build, compose, and decompose complex two- and three-dimensional objects, which they can do with a variety of media including paper-and-pencil sketches, geometric models, and dynamic geometry software. Interpreting or drawing different views of building such as the base floor plan and front and back views, using dot paper can be useful in developing visualization. Students should build three-dimensional objects from two-dimensional representations; draw objects from a geometric description; and write a description, including its geometric properties, for a given object." (p. 237)

According to the *A Research Companion to Principles and Standards for School Mathematics*, "A construct that has been applied extensively to geometric thinking and learning is that of the concept image—a combination of all the mental pictures and properties that have been associated with the concept (Vinner & Hershkowitz, 1980). Students often use concept images rather than definitions of concepts in their reasoning." (p. 155)