Grade 7 Mathematics, Quarter 3, Unit 3.1

Generating Equivalent Expressions

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned

- Understand how to use variables to represent quantities.
- Understand how to use and identify the sequence of order of operations to simplify expressions.
- Formulate and simplify expressions to model real-world problems.
- Understand how to rewrite expressions in different forms.
- Use properties of operation (i.e., Commutative Property, Associative Property, Distributive Property) to help solve equations and inequalities.
- Add, subtract, factor and expand linear expressions with rational number coefficients.
- Rewrite and interpret algebraic expressions to better understand the their meaning.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem and restate it in their words.
- Check answers to problems using a different method.
- Consider different problems to gain insight into its solution.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem solving.
- Use varied representations and approaches when solving problems.
- Know and flexibly use different properties of operations.

Attend to precision.

- Be precise in defining variables.
- Communicate their understanding of mathematics to others.
- Use clear definitions and state the meaning of the symbols they choose.

Look for and make use of structure.

- Students can see complicated things (such as algebraic expressions) as either single objects or as being composed of several objects.

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated, and look both for general methods and shortcuts.
- Maintain oversight of the process while attending to details (use caution with factoring and expanding expressions).

- Evaluate the reasonableness of their intermediate results.

Essential questions

- How do you use variables to represent quantities in real-world situations?
- What do you need to know to write an expression?
- How do you know when two expressions are equivalent?
- How do you use the order of operations to rewrite an expression?
- How do the context of the problem help you write an expression?
- How do the properties of operations help you rewrite an expression?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations	**7.EE**

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Grade 6 students used the order of operations and properties of operations, including the distributive property, to transform simple expressions. They used only whole number coefficients and whole number exponents within simple expressions. They also begin to identify equivalent expressions. Earlier in grade 7, unit 2.1 addressed all operations with rational numbers.

Current Learning

In grade 7, students add, subtract, factor, and expand linear expressions with rational coefficients. Students are able to rewrite and interpret algebraic expressions to gain meaning into real-world problems. It is critical that students continue utilizing their understanding of rational numbers as they formulate expressions and equations in one variable to solve problems.

Future Learning

In grade 8, students will be working with linear equations with rational number coefficients and will transform equations into simpler forms. This may include expanding expressions, factoring expressions, and combining like terms.

Additional Findings

According to *Principles and Standards for School Mathematics* states “patterns of student errors in their use of variables occur … when the child experiences discomfort attempting to handle an algebraic expression which represents a process that [he or she] cannot carry out … for example, in arithmetic, a final answer is always in terms of a single numeric answer, whereas an algebraic ‘answer’ may express a process, such as $2+3a$… An algebraic expression, such as $2 + 3a$, represents both the process by which the computation is carried out and also the product of that process. They attribute these difficulties to traditional teaching in which rules for manipulation are given primary if not sole emphasis, with the ‘forlorn hope’ that understanding will follow” (p. 139)

Common Core State Standards Progressions 6–8, Expressions and Equations: An expression is a phrase in a sentence about a mathematical or real-world situation. As with a facial expression, however, you can read a lot from an algebraic expression (an expression with variables in it) without knowing the story behind it, and it is a goal of this progression for students to see expressions as objects in their own right, and to read the general appearance and fine details of algebraic expressions… In grades 6–8, students start to use properties of operations to manipulate algebraic expressions and produce different but equivalent expressions for different purposes. This work builds on their extensive experience in K–5 working with the properties of operations in the context of operations with whole numbers, decimals and fractions.

Grade 7 Mathematics, Quarter 3, Unit 3.2

Solving Word Problems Algebraically

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Solve multistep real-life and mathematical problems with positive and negative rational numbers in any form.
- Apply properties of operations (e.g., Distributive Property).
- Use mental computation and estimation strategies to determine the reasonableness of answers.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are rational numbers.
- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are rational numbers.
- Graph the solution set of the inequality and interpret it in the context of the problem.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of the problem and restate it in their words.
- Analyze givens and relationships (problem situations, inequalities, equations).
- Check their answers to problems using a different method (using estimation and mental computations).

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Use varied representations and approaches when solving problems.
- Know and flexibly use different properties of operations.

Look for and make use of structure.

- Look for, develop, generalize and describe a pattern orally, symbolically, graphically and in written form.
- Apply and discuss properties.
- Students can see complicated things (such as some algebraic expressions) as either single objects or being composed of several objects.

Essential questions

- What does it mean to say that two quantities are equal?
- What processes would you use when solving a multistep equation?
- Why would graphing the solution(s) to an inequality be helpful in solving a problem?
- How could you use the properties of operations be useful in solving an equation?
- How would you use mental computation or estimation skills aid in checking your answer to a problem?
- What is the process for solving a multiplication equation with a rational coefficient?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations	7.EE

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

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Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Clarifying the Standards

Prior Learning

In grade 6, students worked on solving one-variable equations and inequalities of the form $x + p = q$, $px = q$, $x > c$, and $x < c$, where p and q were nonnegative rational numbers. Students recognized that inequalities have infinitely many solutions and represented those inequalities on number line diagrams. In unit 3.4 (grade 7), students built their understanding of variables to represent real-life situations and interpreted expressions to gain meaning. Students added, subtracted, factored, and expanded linear expressions with rational coefficients in unit 3.4 also.

Current Learning

In grade 7, using different number formats, students solve real-life and mathematical problems using tools strategically and determine the reasonableness of their answers through mental computation and estimation strategies. Students use variables to represent unknown quantities in real-world and mathematical problems, as well as write and solve simple equations and inequalities through reasoning about the quantities. Students solve word problems and equations and/or inequalities in which the equations are in the form of $px + q = r$ and $p(x+q) = r$ and inequalities are of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Students graph the solution set of inequalities and interpret their findings based on the context of the problem.

Future Learning

In grade 8, students will solve linear equations in one variable and will have the ability to give examples of linear equations (in one variable) that have one solution, no solution, or infinitely many solutions through simplifying into equivalent equations. Students will solve linear equations with rational number coefficients, where they will use the distributive property to expand expressions and collect like terms. Students will also be solving pairs of simultaneous linear equations using graphs, algebra, and problem situations. Students will continue their reasoning with inequalities in algebra in high school.

Additional Findings

According to the *Common Core State Standards Progressions for Grades 6 – 8, Expressions and Equations*, "By grade 7, students start to see whole numbers, integers, and positive and negative fractions as belonging to a single system of rational numbers, and they solve multi-step problems involving rational numbers presented in various forms. Students use mental computation and estimation to assess the reasonableness of their solutions…The steps in solving [an] equation mirror the steps in the numerical solution. As problems get more complex, algebraic methods become more valuable" (pp. 8–9).

Principles and Standards for School Mathematics states that, "In the middle grades, students should work more frequently with algebraic symbols than in lower grades. It is essential that they become comfortable in relating symbolic expressions containing variables to verbal, tabular, and graphical representations of numerical and quantitative relationships. Students should develop an initial understanding of several different meanings and uses of variables through representing quantities in a variety of problem situations. They should connect their experiences with linear functions to their developing understandings of proportionality" (p. 223).

Grade 7 Mathematics, Quarter 3, Unit 3.3

Investigating Chance

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand that probability falls between zero (impossible) and one (certainty).
- Approximate probability based on observations and data collection.
- Predict the number of positive outcomes given the theoretical probability (know the difference between theoretical and experimental probabilities).
- Develop a probability model with likely or unequally likely outcomes and use the model to determine probabilities of events.
- Compare probability of a model (theoretical) to observed (experimental) frequencies.

Mathematical practices to be integrated

Model with mathematics.

- Analyze mathematical relationships to draw conclusions using appropriate methods of finding probability.
- Interpret results and draw conclusions from the data that relate to the questions posed.

Use appropriate tools strategically.

- Use tools such as tables, lists, and manipulatives to solve, explore, compare, and visualize problems and to deepen understanding of probability.

Attend to precision.

- Display data accurately and use appropriate labels/units.
- Communicate understanding of math to others.

Essential questions

- How do you find the theoretical probability of an event?
- How do you find the experimental probability of an event?
- How can you make a prediction about the probability of an event?
- What situations would produce a probability of zero, one, or between zero and one?
- When might a situation produce an outcome that is not equally likely?

Written Curriculum

Common Core State Standards for Mathematical Content

Statistics and Probability	7.SP

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times*.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In prior grades, probability has not been introduced. In grades 4 and 5, students developed ideas related to the concept of ratios. This is an important pre-skill that will be utilized when understanding and reporting probability of events.

Current Learning

In grade 7, students understand that the probability of a chance event is a number between 0 and 1, where 0 represents an unlikely event and 1 is a likely event. They collect data and approximate probabilities based on models (theoretical probability). They predict a probability based on observations in an experimental situation (experimental probability). Students develop a probability model and use it to find probabilities of events with equal and unequal outcomes.

Future Learning

In grade 7, this unit is followed by unit 2.3 *Compound Events*. Students will use and develop probability models by finding probabilities of compound events using organized lists, tables, tree diagrams, and simulation. After grade 7, high school students will use probability to evaluate outcomes of decisions.

Additional Findings

According to *Principles and Standards for School Mathematics,* "teachers should give middle-grade students numerous opportunities to engage in probabilistic thinking about simple situations from which students can develop notions of chance. They should use appropriate terminology in their discussions of chance and use probability to make predications and test conjectures" (pp. 253–254).

According to the *Common Core State Standard Progressions for 6–8 Statistics and Probability*, "In grade 7, students move from concentrating on analysis of data to production of data, understanding that good answers to statistical questions depend upon a good plan for collecting data relevant to the questions of interest. Because statistically sound data production is based on random sampling, a probabilistic concept, students must develop some knowledge of probability before launching into sampling. Their introduction to probability is based on seeing probabilities of chance events as long-run relative frequencies of their occurrence, and many opportunities to develop the connection between theoretical probability models and empirical probability approximations" (p. 2).

Grade 7 Mathematics, Quarter 3, Unit 3.4

Compound Events

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Find sample space for a compound event using lists, tables, tree diagrams, and simulation.
- Identify outcomes in a sample space of a compound event.
- Design and use a simulation to generate frequencies for compound events.
- Understand that the probability of a compound event is still a number between 0 and 1 inclusive.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Choose appropriate organizational tools such as organized lists or tree diagrams and use them to identify outcomes in a sample space.

Model with mathematics.

- Identify important quantities in a practical situation (choosing probabilities that are appropriate factors in a given situation)
- Identify important quantities and map their relationships using lists and diagrams.

Look for and make use of structure.

- Look for, develop, generalize, and describe a pattern orally, symbolically, graphically, and in written form.
- Step back for an overview after simulation and make generalizations based on results.
- In designing a simulation to represent an experiment, understanding that one simulation may be better suited for an experiment than another.

Essential questions

- How do you determine the total number of outcomes of a compound event from a table?
- How do you determine the total number of outcomes of a compound event using lists?
- How do you determine the total number of outcomes of a compound event from a tree diagram?
- How do you determine the total number of outcomes of a compound event using a simulation?
- Which situations lend themselves to compound events?
- In which real-world situations may we utilize a simulation rather than conducting the actual experiment?

Written Curriculum

Common Core State Standards for Mathematical Content

Statistics and Probability	7.SP

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students were first exposed to probability in grade 7. In the previous unit, *Investigating Chance*, students learned that the probability of an event is expressed as a number between zero and one. They learned the difference between theoretical and experimental probability and they used probability models to determine the probability of events.

Current Learning

Seventh-grade students find the sample space for a compound event using lists, tables, and tree diagrams. They identify outcomes in the sample space of a compound event. They design and use a simulation to generate frequencies for compound events. Students understand that the probability of a compound event is still between 0 and 1. This is a newly investigated concept and is not a major focus in grade 7.

Future Learning

In high school, students will use their understanding of probability to interpret data and compute outcomes in real-world problem-solving situations. Students will use their probability work from the middle grades as a basis for statistical inference. They will be using probability to build an understanding of normal distributions, standard deviation, conditional probability, and problems involving expected value.

Additional Findings

According to *Principles and Standards for School Mathematics,* students will compute probabilities for simple compound events using such methods as organized lists, tree diagrams, and area models. Students also need to develop their probabilistic thinking by frequent experience with actual experiments. “Many can be quite simple. For example, students could be asked to predict the probability for various outcomes of flipping two coins sixty times… Then they could discuss whether the results of the experiment are consistent with their predictions. If students are accustomed to reasoning from and about data, they will understand that discrepancies between predictions and outcomes from a large and representative sample must be taken seriously” (pp. 248, 254).

The book also states, “Although the computation of probabilities can appear to be simple work with fractions, students must grapple with many conceptual challenges in order to understand probability. Misconceptions about probability have been held not only by many students but also by many adults (Konold 1989). To correct misconceptions, it is useful for students to make predictions and then compare the predictions with actual outcomes” (pg. 254).