Grade 7 Mathematics, Quarter 2, Unit 2.1

Proportional Reasoning with Rates

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Compute unit rates associated with ratios of fractions.
- Compute unit rates including ratios of lengths and areas.
- Compute unit rates among other measurable quantities measured in like units or different units.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of the problem.
- Consider similar problems (whole numbers vs. fractions to gain insight into its solutions).
- Continually asks, "Does this make sense?" when calculating unit rates.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Consider the units involved. Specify appropriate units of measure when reporting rates.
- Converting a problem situation to a number format to find a solution.

Attend to precision.

- Specify appropriate units of measure when reporting rates.
- Calculate accurately and efficiently.
- Express numerical answers with a degree of precision appropriate to the problem context.

Essential questions

- What is a ratio and how do you recognize a ratio in a real-world or mathematical situation?
- How do you recognize a unit rate and what information does it provide?
- What are two different strategies you could use to find the unit rate when given a ratio, such as a ratio of fractions?
- How can you label the quantities in your problem to clarify your process and the meaning of your answer?
- Why does division make sense in this situation?
- Why is the quotient of two positive fractions larger than the dividend? Use a model or drawing to justify why your answer makes sense.

- How many different unit rates can be written for a given ratio? What does each unit rate mean? Give an example to explain your response.
- Given a unit rate in a problem situation, how can you determine equivalent rates?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System **7.NS**

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 6, students learned about unit rates and used appropriate math language to describe rates and ratios in problem situations.

Current Learning

In grade 7, students compute unit rates associated with ratios containing fractions. They extend their understanding of ratios as they incorporate them into problem solving situations. The unit reinforces prior understanding of unit rates with whole number components and extends their learning to fractional components. Students are being exposed to a variety of problem situations through the math practices listed.

Future Learning

In a future 7th grade unit, students will identify the unit rate in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. They will understand that the unit rate is a measure of steepness on a line graph that is referred to as the slope of the line. In grade 8, students will use their understanding of unit rate as slope to graph proportional relationships and compare two proportional relationships represented in different ways.

Additional Findings

According to *Principles and Standards of School Mathematics*, "Attention to developing flexibility in working with rational numbers contributes to students' understanding of, and facility with, proportionality. Facility with proportionality involves much more than setting two ratios equal and solving for a missing term. It involves recognizing quantities that are related proportionally and using numbers, tables, graphs, and equations to think about the quantities and their relationship. Proportionality is an important integrative thread that connects many of the mathematics studied in grades 6 to 8." (p. 217)

Teachers need to be aware that students struggle with operations related to fractions, especially division. This will directly impact their understanding of ratios and rates. To facilitate students understanding of division of fractions, teachers can choose to help students "understand the division of fractions by building on what they know about the division of whole numbers. If students understand the meaning of division as repeated subtraction, they can recognize that 24 ÷ 6 can be interpreted as 'How many sets of 6 are there in a set of 24?'" (p. 219)

Grade 7 Mathematics, Quarter 2, Unit 2.2

Proportional Reasoning with Percents

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use proportional relationships to solve multistep percent problems involving real-world application and math problems.
- Use proportional relationships to solve multistep ratio problems involving real-world applications and math problems.
- Solve multistep problems involving simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Identify and execute appropriate strategies to solve the problem. Determine whether a quantity should be added or subtracted to find a solution (for example, markup vs. markdown).
- Check answers using a different method, and continually ask, "Does this make sense?"
- Consider similar problems to gain insight into solutions using estimation or benchmarks.

Reason abstractly and quantitatively.

- Make sense of quantities and their proportional relationships in problem situations.
- Use varied representations and approaches when solving problems. (i.e., can use proportions, percent equations, formulas)
- Consider units involved. All solutions need to have appropriate labels.

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Identify important quantities in a practical situation placing them into appropriate variable positions in an equation.
- Interpret their mathematical results. (i.e., which is the better buy?)

Essential questions

- How do you know this relationship is proportional?
- What proportional relationship will help you solve this problem? How can you verify you have recorded your proportional relationship correctly?
- How are your ratios related?
- What information does a percent provide? What are other ways to represent a percent?
- Describe your strategy for finding the percentage of a number.
- How can you use a unit rate to help solve this problem?
- What strategies or models can be used to determine if your solution is reasonable?
- How can you represent a percent of increase or decrease? How can you represent a markup and markdown? Give examples to explain your response.

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships **7.RP**

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 6, students used ratios and rates to solve real-world problems and solved problems that required them to find the whole, given a part and a percent.

Current Learning

Grade 7 students solve multistep ratio and percent problems. These problems include simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error. This is a critical area of grade 7 learning based on the CCSS. Students are reinforcing the concept of finding the whole, given the part and the percent. All other concepts in this unit are at the developmental stage and are practiced and mastered.

Future Learning

Students will not be addressing the concepts in this unit in future grades. Therefore, these topics must be mastered by the end of grade 7.

Additional Findings

In *Principles and Standards of School Mathematics,* we are reminded "middle grade students should see mathematics as an exciting, useful, and creative field of study. As they enter adolescence, students experience physical, emotional, and intellectual changes that mark the middle grades as a significant transition point in their lives. During this time, many students will solidify conceptions about themselves as learners of mathematics—about their competence, their attitude, and their interest and motivation. These conceptions will influence how they approach the study of mathematics in later years, which will in turn influence their life opportunities." (p. 211)

According to *Curriculum Focal Points,* "Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single- and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease." (pp. 36–37)

Grade 7 Mathematics, Quarter 2, Unit 2.3

Analyzing Proportional Relationships with Equations and Graphs

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Write equations that represent proportional relationships (in the form of $y = mx$).
- Explain what the ordered pair (x, y) represents in a contextual situation that illustrates a constant rate of change, paying close attention to the points (0, 0) and $(1, r)$, where r represents the unit rate.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them**.**

- Explain the relationships between equations, verbal descriptions, tables, and graphs.
- Explain the meaning of a problem or a given ordered pair.

Reason abstractly and quantitatively.

- Flow between contextual and non-contextual situations during problem solving and make meaning of numbers and symbols.
- Know and flexibly utilize different representations of a proportional relationship (i.e., verbal descriptions, equations, and graphs).
- Attend to the meaning of quantities.

Use appropriate tools strategically**.**

- Use tools to solve, explore, compare, and visualize problems and to deepen knowledge/understanding.

Essential questions

- How do you represent a proportional relationship with an equation?
- How would you explain all the possible outcomes of a proportional relationship?
- Why does a proportional relationship result in a graph of a line?
- What do the ordered pairs (0, 0) and $(1, y)$ represent on a graph of any problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.2 Recognize and represent proportional relationships between quantities.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 5, students began to use a coordinate plane and coordinates to identify a location. They represented real-world and mathematical problems by graphing points in the first quadrant of the graph. In grade 6, students developed an understanding of the concept of a unit rate. They solved unit rate problems, including those involving unit pricing and constant rate of speed. They have also made tables of equivalent ratios that relate two quantities, filled in missing values in a table, plotted pairs of values on the coordinate plane and used tables to compare ratios.

Current Learning

In grade 7, students recognize proportional relationships through using equations to represent real-life situations understanding that when one quantity changes the other quantity varies directly ($y = mx$). In relation to this, students interpret the meaning of a coordinate point in a problem situation. (For example, what does the point (x, y) mean on a graph?) Also, grade 7 students focus on the relevance of (0, 0) and (1,r), where r represents the unit rate. Using different number formats, students will solve real-life and mathematical problems strategically and determine reasonableness of answers.

Future Learning

In grade 8, students will interpret equations in the form of $y = mx + b$ as linear functions that form a straight line when graphed. They will give examples of non-linear functions and will be able to justify their reasoning using ordered pairs. Students will also describe quantitatively the functional relationship between two quantities through analyzing a graph and be able to sketch the graph of that function. Students will construct functions to model a linear relationship between two quantities. They will be able to determine and to interpret the rate of change and initial value of a function from a description of a relationship or from two (x, y) values, a graph, or a table of values.

Additional Findings

According to *Principles and Standards for School Mathematics,* "Most students will need extensive experience in interpreting relationships among quantities in a variety of problem contexts before they can work meaningfully with variables and symbolic expressions. An understanding of the meanings and uses of variables develops gradually as students create and use symbolic expressions and relate them to verbal, tabular, and graphical representations. Relationships among quantities can often be expressed symbolically in more than one way, providing opportunities for students to examine the equivalence of various algebraic expressions" (pp. 225–226).

In this grade level, students tend to make errors by placing numbers in incorrect locations within proportions, especially when the order in which quantities are stated in a problem are switched. (*CCSS Progressions 6 – 7 Ratios and Proportional Relationships*, p. 9)

Grade 7 Mathematics, Quarter 2, Unit 2.4

Applying Proportional Reasoning to Geometry

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Compute actual length and area from a scale drawing.
- Reproduce a scale drawing with a different scale.
- Solve problems involving scale drawing of geometric figures.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use tools when solving a mathematical problem to deepen the understanding of concepts (e.g. paper and pencil, physical models, geometric construction and measurement devices, graph paper, or calculators).

Attend to precision.

- Specify units of measure and use a scale to create visual representations.

Model with mathematics.

- Identify important quantities in a practical situation with the use of scale drawings.
- Apply the mathematics they know (proportional relationships) to solve problems arising in everyday life, society and the workplace.

Look for and make use of structure.

- Students step back for an overview and shift perspective.
- Describe patterns in different ways: orally, symbolically, written form.
- Apply and discuss properties.

Look for and express regularity in repeated reasoning.

- Evaluate the reasonableness of their results.

Essential questions

- How does geometry help us describe real-world objects?
- What effect does scale factor have on area?
- What effect does scale factor have on linear measures and perimeter?
- What effect does scale factor have on angle measures?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students in grade 6 have drawn polygons in the coordinate plane to prepare for constructions. In a prior unit in grade 7, students prepared for their work with scale drawings by using proportional reasoning with rates and percents.

Current Learning

Students in grade 7 solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. This is a critical area and will be assessed in grade 7.

Future Learning

In grade 8, students will understand that two-dimensional figures can be transformed into either two-dimensional or similar figures through a series of translations, rotations, and reflections.

Additional Findings

According to *Common Core State Standards Progressions for 6-7, Ratios and Proportional Relationships*, "Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life. Students use ratios in geometry and in algebra when they study similar figures and slopes of lines, and later when they study sine, cosine, tangent and other trigonometric ratios in high school. (p. 2)

According to *Principles and Standards for School Mathematics*, "Middle-grades students also need experience in working with congruent and similar shapes. From their earlier work, students should understand that congruent shapes and angles are identical and can be "matched" by placing on atop the other. Students can begin with an intuitive notion of similarity; similar shapes have congruent angles but not necessarily congruent sides. In the middle grades, they should extend their understanding of similarity to be more precise, noting, for instance, that similar shapes "match exactly when magnified or shrunk" or that their corresponding angles are congruent and their corresponding sides are related by a scale factor. Students can investigate congruence and similarity in many settings, including art, architecture, and everyday life." (p. 234)