Grade 7 Mathematics, Quarter 1, Unit 1.1

Positive Rational Number Operations

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve real-world problems across all operations with positive rational numbers including complex fractions.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze given information to develop possible strategies for solving the problem. Identify operation, problem-solving strategies, and numbers involved.
- Identify and execute appropriate strategies to solve the problem. Utilize positive rational number operations.
- Check their answers using a different method and continually ask, ”Does this make sense?”

Construct viable arguments and critique the reasoning of others.

- Justify conclusions, communicate them to others, and respond to the arguments of others.
- Use correct logic and reasoning. Read for understanding.
- Seek to understand alternative approaches suggested by others and, as a result, adopt better approaches.

Attend to precision.

- Communicate their understanding of mathematics to others using words and math processes. Students should be able to write their answers in complete sentences and explain how they got their answer.
- Use an appropriate degree of precision (for example, when dealing with money, round to the first two decimal place values).

Essential questions

- What is the question being asked? What information is needed to answer the question?
- Predict the type and size of answer that would seem reasonable for this problem situation. How did you determine a reasonable estimate?
- Justify why the operation you selected makes sense for this problem.
- What steps could you take to verify your solution is correct?
- What form of number (decimal or fraction) is the most efficient to use for this calculation? Justify your choice.
- Explain how place value and the properties of operations (e.g., decomposing and composing numbers) are used in your computations.
- What real-world situations would cause you to divide a fraction by a fraction?
- Predict how the size of the quotient will compare to the dividend. Give mathematical reasons to support your prediction.

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System **7.NS**

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Beginning in grade 3, students were introduced to fractions. In grades 4 and 5, they learned to work with equivalent fractions, comparing fractions, and the four operations with fractions. Also in grade 5, students were introduced to decimals and used them in all four operations. In grade 6, students extended their operations knowledge to solve word problems involving all positive fractions. They were introduced to the concept of rational numbers, both positive and negative, and were able to write, interpret, and explain situations that order rational numbers in real-world contexts.

Current Learning

In grade 7, students have clear understanding of all forms of positive rational numbers. They solve real-world and mathematical problems using addition, subtraction, multiplication, and division with positive rational numbers. Students extend their knowledge of manipulating fractions to simplifying and working with complex fractions. It is critical for students to master their understanding of working with rational numbers at this grade level.

Future Learning

In grade 8, students will learn about irrational numbers and how to approximate them. They will continually apply their understanding of operations with rational numbers to solve algebraic equations and simplify expressions.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics states, "… we find it profitable to point out that understanding the rational number system, where 'rational numbers' is used as mathematicians use it, is so far beyond the grasp of school students that curriculum and instructional designers must be clear on what they mean by 'fractions' and 'rational numbers' so they avoid designing for incoherent learning goals." (p. 98)

According to *Principles and Standards For School Mathematics*, "the study of rational numbers in the middle grades should build on students' prior knowledge of whole-number concepts and skills and their encounters with fractions, decimals, and percents in lower grades and in everyday life. Students' facility with rational numbers and proportionality can be developed in concert with their study of many topics in the middle grades curriculum. For example, students can use fractions and decimals to report measurements, to compare survey responses from samples of unequal size … In middle grades, students should expand their repertoire of meanings, representations, and uses for nonnegative rational numbers." (pp. 215–216)

Grade 7 Mathematics, Quarter 1, Unit 1.2

Understanding Positive and Negative Rational Number Operations

Overview

Number of instructional days: 20 (1 day = 45–60 minutes)

Content to be learned

- Apply and extend understanding of the four operations with respect to both positive and negative rational numbers.
- Represent addition and subtraction of signed rational numbers on both horizontal and vertical number lines.
- Describe real-life situations where numbers come together to make zero.
- Understand the concept of absolute value and its implications when adding and subtracting.
- Interpret sums, differences, products, and quotients of rational numbers by describing real-world contexts.
- Apply properties of operations as strategies (commutative, associative) to add, subtract, multiply, and divide rational numbers.
- Understand that multiplication of signed rational numbers is the same as repeated addition where the same integer rules apply.
- Know that every quotient of integers is a rational number, provided that the divisor is not zero.
- Convert a rational number to its decimal form using long division.
- Know that the decimal form of a rational number terminates or eventually repeats.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Consider similar problems to gain insight into a solution.
- Make conjectures about solutions such as the result of a fraction divided by a fraction (how the quotient is larger).
- Choose a correct strategy and apply it to the rational number operation.
- Check answers to problems using a different method.

Attend to precision.

- Strive for accuracy/use an appropriate degree of precision.
- Communicate understanding of rational numbers to others.

Look for and make use of structure.

- Look for, develop, generalize, and describe a pattern orally, symbolically, graphically, and in written form.
- Look closely to discern a pattern or structure.

Look for and express regularity in repeated reasoning.

- Notice the regularity that might lead to a general formula or shortcut.
- Evaluate the reasonableness of intermediate results.

Essential questions

- Why is it important to understand properties and operations involving signed rational numbers?
- How do you demonstrate addition and subtraction of signed rational numbers using number lines?
- What does absolute value mean?
- What does the commutative property state about the addition and multiplication of rational numbers?
- What are the similarities and differences between addition and subtraction of rational numbers?
- When is the sum of two numbers equal to zero?
- What are some real-world situations when you need to apply one or more of the four basic operations of rational numbers?
- Why might it be appropriate to convert a rational number to its decimal form?
- How would you convert a rational number to its decimal form?
- What are the characteristics of a rational number?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System **7.NS**

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grades 4 and 5, students were introduced to operations with fractions. All four operations with decimals were introduced in grade 5. In grade 6, students calculated decimals fluently. They understood the relationship between multiplication and division to explain why the procedures for dividing fractions make sense. They extended previous understanding of ordering numbers to include the full system of rational numbers (negative integer numbers in particular). They understood the concept of absolute value of rational numbers and were able to locate points on a coordinate plane.

Current Learning

In grade 7, students extend learning from grade 6 and grade 7 (unit 1.4) to include positive and negative rational numbers. Properties of operations (distributive, commutative, additive inverse) help students to add, subtract, multiply and divide positive and negative rational numbers. They reinforce understanding of operations with positive rational numbers, but students need to develop their understanding of the four operations with negative rational numbers. All students convert rational numbers to decimals using long division. They should know that the decimal form of a rational number either repeats or terminates.

Future Learning

In unit 3.4 of grade 7, students will apply properties of operations to help them add, subtract, factor, and expand linear expressions with rational coefficients. In grade 8, students will move from rational to irrational numbers. They will understand that every number has a decimal expansion. They will use rational approximations of irrational numbers and locate them on a number line.

Additional Findings

According to *Principles and Standards for School Mathematics,* "in grades 6–8, students should deepen their understanding of fractions, decimals, percents, and integers, and they should become proficient in using them to solve problems … The study of rational numbers in the middle grades should build on students' prior knowledge of whole-number concepts and skills and their encounters with fractions, decimals, and percents in lower grades and in everyday life." (p. 215)

Grade 7 Mathematics, Quarter 1, Unit 1.3

Representing Ratios and Unit Rates

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Determine whether two quantities are in a proportional relationship (using tables or graphs to determine a constant rate of change).
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations. (Are two quantities proportional?)
- Use varied representations including tables, graphs, equations, diagrams, and verbal descriptions.

Construct viable arguments and critique the reasoning of others.

- Understand and use prior learning (make connections to/with prior units 1.2 and 1.3 and grade 6 ratio and unit rate work) in constructing arguments.
- Seek to understand alternative approaches suggested by others and as a result, adopt better approaches (identifying the constant of proportionality might be found easier through another approach than the original one chosen).
- Justify their conclusions, communicate them to others, and respond to the arguments of others (explain why two quantities may or may not be proportional; explain how the constant of proportionality is determined).

Attend to precision.

- Strive for accuracy (when determining proportional relationships, tables and/or graphs need to be accurate to make an informed and accurate decision).
- Communicate their understanding of mathematics to others (tables, graphs, equations, diagrams, and verbal descriptions must be accurate and clear).

Essential questions

- How can you determine whether two ratios are equivalent?
- How do you know if two quantities have a proportional relationship?
- How can you identify the constant of proportionality through tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships?

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships **7.RP**

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 6, students used their understanding of fractions from previous grades to develop the concepts of ratio and unit rate. They also used their ratio and rate reasoning to solve mathematical problems using a variety of methods (e.g., tables, tape diagrams, double number line diagrams or equations). In grade 7 unit 1.2, students computed unit rates with ratios of fractions.

Current Learning

Students extend their understanding of ratios and develop understanding of proportionality. They use tables or graphs to represent proportional relationships, looking for constant rates of change. They identify unit rates through several modes: tables, graphs, equations, diagrams, and verbal descriptions.

Future Learning

In grade 8, students will graph proportional relationships and interpret the results as slope. Students will compare graphs and equations of proportional relationships.

Additional Findings

According to *Principles and Standards for School Mathematics,* "Curricular focus and integration are also evident in the proposed emphasis on proportionality as an integrative theme in the middle-grades mathematics program. Facility with proportionality develops through work in many areas of the curriculum, including ratio and proportion, percent, similarity, scaling, linear equations, slope, relative-frequency histograms, and probability. The understanding of proportionality should also emerge through problem solving and reasoning, and it is important in connecting mathematical topics and in connecting mathematics and other domains such as science and art" (p. 212). In other words, students will spend a good portion of their time in the middle grades working on proportionality and making connections among the different math strands."

Also, "the study of patterns and relationships in the middle grades should focus on patterns that relate to linear functions, which arise when there is a constant rate of change. Students should solve problems in which they use tables, graphs, words, and symbolic expressions to represent and examine functions and patterns of change" (*Principals and Standards for School Mathematics*, p. 223). Students should be able to use a variety of representations and decide which represent constant rates of change.