Kindergarten Mathematics, Quarter 3, Unit 3.1

Subtracting Whole Numbers Within 10

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Show subtraction using real-world situations.
- Subtract within 10, using strategies such as objects, drawings, sounds, or words to represent the problem.
- Explore and solve subtraction word problems with whole numbers from 0–10.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.
- Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"

Model with mathematics.

- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation.
- They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Essential questions

- How did you solve this problem?
- How can you prove your answer?
- How can you use your fingers, objects, or drawings to help you find the answer?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent ~~addition and~~ subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, ~~or equations~~.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve ~~addition and~~ subtraction word problems, ~~and add~~ and subtract within 10, e.g., by using objects or drawings to represent the problem.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of

the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

According to the Rhode Island Early Learning Standards, students entering kindergarten may "begin to use numbers and counting as a means for solving problems and they may use words such as more than, less than and add/subtract to express some number concepts."

According to *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), "students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting." (p. 8)

Current Learning

Students investigate subtraction of whole numbers within 10 by solving problems that involve taking apart or taking from. They use real-life situations to show subtraction and they use strategies such as drawings, objects, and fingers to represent problems. In Quarter 4, they incorporate equations into their work with addition and subtraction.

This unit is taught at the developmental level.

Routines established for counting objects with one-to-one correspondence, composing/decomposing numbers, and adding numbers are expected to continue for this unit.

Future Learning

In grade 1, students work will with addition and subtraction problems within number 20 using up to three whole numbers. They will relate counting to addition and subtraction (e.g. counting on 2 to add 2). Students will determine the unknown number in an addition or subtraction equation. They will incorporate place value into addition problems by adding two-digit and one-digit numbers, and adding a two-digit number and a multiple of 10, using models, drawings or other strategies.

Additional Findings

Principles and Standards for School Mathematics states that "an understanding of addition and subtraction can be generated when young students solve 'joining' and 'take away' problems by directly modeling the situation or by using counting strategies, such as counting on or counting back. Students develop further understandings of addition when they solve missing addend problems that arise from stories or real situations" (p. 83).

According to *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), "students act out adding and subtracting situations by representing quantities in the situation with objects, their fingers, and math drawings. To do this, students must mathematize a real-world situation, focusing on the quantities and their relationships rather than non-mathematical aspects of the situation. Situations can be acted out and/or presented with pictures or words. Math drawings facilitate reflection and discussion because they remain after the problem is solved. These concrete methods that show all of the objects are called Level 1 methods." (p.8)

In addition, it states that "students learn and use mathematical and non-mathematical language, especially when they make up problems and explain their representation and solution." (p. 8)

Kindergarten Mathematics, Quarter 3, Unit 3.2

Developing Number Sense and Labeling Sets

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Count orally from 1-100 by ones and tens, saying the number names in standard order.
- Count orally, pairing each object with one and only one number (1-1 correspondence).
- Count the objects in a set and state that the last number name said tells how many objects are in the set.
- Represent a set of up to 20 objects with a written numeral.
- Understand that each number said when counting refers to a quantity that is one larger.

Mathematical practices to be integrated

Reason abstractly and quantitatively

- Mathematically proficient students make sense of quantities and their relationships in problem situations.
- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Look for and express regularity in repeated reasoning.

- Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.
- The repetitive nature of counting by ones and tens generalizes understanding of the number system.
- As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details.

Essential questions

- As you count from 1–100 (by 1's or by 10's), how do you know if you are counting correctly?
- How do you know whether it would be easier to count groups of objects by tens or by ones?
- How can you show which set of objects matches this numeral?
- How is a set of (9) objects different from a set of (8) objects?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality K.CC

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

c. Understand that each successive number name refers to a quantity that is one larger.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Children may enter kindergarten with an awareness of numbers and an ability to count them. According to the Rhode Island Early Learning Standards (www.ride.ri.gov/els/resources.asp), beginning kindergarten students may begin to "use numbers and counting as a means for solving problems, predicting, and measuring quantity." They may be able to "use one-to-one correspondence in counting objects and matching groups of objects and begin to associate a number of objects with names and symbols for numbers.

Current Learning

Students learn to count to 100 by ones and tens. They learn how to write the numerals from 0-20 and represent groups of objects with written numerals. Students understand that when counting, the next number counted refers to a quantity that is one larger.

Instruction at this unit is at the developmental level.

Routines established for rote counting and counting objects with one-to-one correspondence are expected to continue for this unit.

Future Learning

In grade 1, students will count rotely to 120 starting at any number less than 120. They will read and write numerals and represent groups of objects with written numerals. They will relate counting to addition and subtraction, i.e. counting on by 2 to add 2.

Additional Findings

According to *Progression for the Common Core State Standards for Mathematics* - draft (commoncoretools.wordpress.com), "students usually know or can learn to say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects. Students become fluent in saying the count sequence so that they have enough attention to focus on the pairings involved in counting objects. To count a group of objects, they pair each word said with one object."

Also, "understanding that each successive number name refers to a quantity that is one larger is the conceptual start for Grade 1 counting on. Prior to reaching this understanding, a student might have to recount entirely a collection of known cardinality to which a single object has been added."

Kindergarten Mathematics, Quarter 3, Unit 3.3

Identifying, Analyzing, and Comparing 2-D and 3-D Objects

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Correctly name shapes regardless of orientation or overall size.
- Identify shapes as being either two-dimensional or three-dimensional.
- Compare and contrast various two-dimensional shapes in different sizes and orientations and describe their attributes.
- Compare and contrast various three-dimensional shapes in different sizes and orientations and describe their attributes.
- Describe measurable attributes of objects, such as length and weight.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.

Construct viable arguments and critique the reasoning of others.

- Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions.

Essential questions

- How can you describe this shape? (parts, attributes)
- How can you sort these two-dimensional and three-dimensional shapes?
- How do you know if this shape is two-dimensional or three-dimensional?
- How are these two-dimensional shapes alike/different?
- How are these three-dimensional shapes alike/different?
- What can you tell me about the (length, height, width, weight) of each of these objects compared to Object A?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.2 Correctly name shapes regardless of their orientations or overall size.

K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

Measurement and Data K.MD

Describe and compare measurable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

According to the NCTM *Curriculum Focal Points*, children "find shapes in the environment and describe them in their own words. They build pictures and designs by combining two- and three-dimensional shapes, and they solve such problems as deciding which piece will fit into a space in a puzzle." (p. 11)

According to the Rhode Island Early Learning Standards (see the RIDE website), students entering kindergarten may "begin to order, compare, or describe objects according to size, length, height, and weight using standard or non-standard forms of measurement."

Current Learning

Students in kindergarten correctly name shapes regardless of their orientation or overall size. They learn the difference between two-dimensional or three-dimensional shapes. They look for similarities and differences between two- and three-dimensional shapes, and describe their parts and other attributes. Students describe measurable attributes of objects, such as length and weight, and compare two objects to describe attributes of a single object.

Instruction in this unit is taught at the developmental level.

The routines of recognizing and naming two-dimensional shapes as well as reinforcing the practice of directly comparing two objects with a measurable attribute in common, should be reinforced in this unit.

Future Learning

In grade 1, students will compose two- or three-dimensional shapes to create a composite shape and compose new shapes from the composite shape. They will order three objects by length and compare the lengths of two objects by using a third object.

Additional Findings

Principles and Standards of School Mathematics states that "PreK–2 geometry begins with describing and naming shapes. Young students begin to use their own vocabulary to describe objects talking about how they are alike and how they are different. Teachers must help students gradually incorporate conventional terminology into their description of two- and three-dimensional shapes. However, terminology should not be the focus of PreK-2 geometry prorgrams. The goal is that early experiences with geometry lay the foundation for more formal geometry in later years." (p. 97)

Also according to *Principles and Standards of School Mathematics,* "children in PreK-2 begin by comparing and ordering objects using language, such as *longer* and *shorter.*" Also, "children begin to develop an understanding of attributes by looking at, touching, or directly comparing objects. The can compare shoes, placing them side by side, to check with is longer. Adults should help young children recognize attributes through conversation." (p. 103)

Kindergarten Mathematics, Quarter 3, Unit 3.4

Composing/Decomposing to 10 With Recording

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Determine how to separate a number between 1-10 into pairs in more than one way, using drawings, and record the answer (e.g. "circle your answer"). *
- Determine the number that needs to be added to another number between 1-9 to make 10, using drawings, and record the answer (e.g. "circle your answer"). *

**Manipulative will be offered to students to assist in calculations.*

Mathematical practices to be integrated

Model with mathematics.

- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as drawing pictures to solve the problem and circling the answer.
- They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. In the early grades, this may include whole-class discussion, small-group discussion, or paired discussions, eventually leading to independent thinking to determine if the answer is reasonable.

Attend to precision.

- Mathematically proficient students try to communicate precisely to others. They use discussions with others to clarify their own reasoning.
- Students give carefully formulated explanations to each other.

Essential questions

- How can you use counters and a ten-frame to break this number up into of smaller numbers?
- How can you find out what number needs to be added to this number to make (said number)?
- Can you explain how you can use a picture to solve this problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing ~~or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).~~

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing ~~or equation~~.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

According to the Rhode Island Early Learning Standards (www.ride.ri.gov/els/resources.asp), students entering kindergarten may begin to "use numbers and counting as a means for solving problems, predicting, and measuring quantity." They may be able to "use one-to-one correspondence in counting objects and matching groups of objects, and begin to associate a number of objects with names and symbols for numbers."

Current Learning

Kindergarten students match the numbers 0-10 to a set that represents that number using one-to-one correspondence. They learn to take a set of objects from 1-10 and separate it into two smaller pairs in multiple ways and to add a certain number of objects to a given set to make a number up to 10. The level of instruction for these skills should be provided at the reinforcement level.

Students are now expected to record their findings from a composing/decomposing situation using drawings, and will later record using equations. Students are beginning to discuss their thinking in various-sized groupings (e.g. whole-class, small-group, and pairs). The level of instruction for recording should be at the developmental level.

The routines of counting by ones and tens, and counting objects with one-to-one correspondence are appropriate to this unit.

Future Learning

In grade 1, students will use addition and subtraction of up to three whole numbers within 20 to solve problems involving adding to, taking from, putting together, and taking apart. They will use objects, drawings, and equations with a symbol for the unknown number (such as a question mark). In grade 2, students will solve one- and two-step word problems using addition and subtraction within 100.

Additional Findings

According to the *Progressions for the Common Core State Standards in Mathematics* K-5 Operations and Algebraic Thinking (May 2011), "in Put Together /Take Apart situations, two quantities jointly compose a third quantity (the total), or a quantity can be decomposed into two quantities (the addends). This composition /decomposition may be physical or conceptual. These situations are acted out with objects initially and later children move to conceptual mental actions of shifting between seeing the addends and seeing the total (e.g. seeing children or seeing boys and girls, or seeing red and green apples or all the apples)." (p.10)

It also states that "Put Together/Take Apart situations with Both Addends Unknown play and important role in Kindergarten because they allow students to explore various compositions that make each number. This will help students to build the Level 2 embedded number representations used to solve more advanced problem subtypes. Students can find patterns in all of the decompositions of a given number and eventually summarize these patterns for several numbers." (p. 10)