Kindergarten Mathematics, Quarter 2, Unit 2.1

Counting and Counting Forward up to 50

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Count orally from 1–50.
- Say the number names in the standard order when rote counting.
- Count forward by ones from a starting point other than number 1.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- Analyze givens, constraints, relationships, and goals.
- Monitor and evaluate progress and change course if necessary.

Look for and make use of structure.

- Look closely to discern a pattern or structure.

Essential questions

- How can you count from 1–50 with all of the numbers in the right order?
- How can you count correctly when you start at a number other than 1?
- What number comes after _____? How do you know?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality K.CC

Know number names and the count sequence.

K.CC.1 Count ~~to 100~~ by ones ~~and by tens~~.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students may enter kindergarten knowing the counting words and be able to do some basic counting in order from number 1. They may have some prior understanding of what distinguishes a number from a shape or a letter.

Current Learning

Students count orally from 1–50 by ones and they learn the standard order of numbers. Within numbers 1–50, they count forward from a number other than number 1.

Later in the year, students count to 100 by ones and tens.

Instruction in this unit is at the developmental level.

Routines established for oral counting and counting objects with one-to-one correspondence continue in this unit.

Future Learning

In grade 1, students will count to 120, starting at any number less than 120. In grade 2, students will count within 1,000 and skip-count by 5s, 10s, and 100s.

Additional Findings

According to the *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), students usually know or can learn to say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects. Students become fluent in saying the count sequence so that they have enough attention to focus on the pairings involved in counting objects (p. 4).

Kindergarten Mathematics, Quarter 2, Unit 2.2

Extend Understanding of Number by Classifying and Comparing Groups of Objects

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Sort objects into categories.
- Count the objects in each category using correct one-to-one correspondence.
- Order the categories according to the number of objects in each one.
- Compare two objects with a common measurable attribute to see which object has more/less of the attribute, and describe the difference between the two objects.
- Use matching and counting strategies to identify whether one group of objects is greater than, less than, or equal to a second group of objects.
- Compare two written numerals between 1 and 5, due to the abstract nature of this skill.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.

Construct viable arguments and critique the reasoning of others.

- Construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

Attend to precision.

- Communicate precisely to others.
- Use clear definitions in discussion with others and in their own reasoning.
- Calculate accurately and efficiently.
- Express numerical answers with a degree of precision appropriate for the problem context.

Essential questions

- How can you sort these objects?
- How can you find out how many objects are in each group?
- How can you put the groups in order after counting how many objects are in each group?
- How can you describe the difference between these two objects (using attributes such as longer/shorter/taller, heavier/lighter, wider/narrower)?
- What strategies can you use to find out whether Group A has more, less, or the same number of objects as Group B?
- What is the relationship between these two numerals?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data K.MD

Classify objects and count the number of objects in each category.

K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

[3] Limit category counts to be less than or equal to 10.

Describe and compare measurable attributes.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Counting and Cardinality K.CC

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]

[1] Include groups with up to ten objects.

Compare numbers.

K.CC.7 Compare two numbers ~~between 1 and 10~~ presented as written numerals.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students entering kindergarten may be able to recognize similarities and differences among everyday objects. According to the Rhode Island Early Learning Standards (http://www.ride.ri.gov/els/doc.asp), they may begin to order, compare, or describe objects according to size, length, height, and weight using standard and non-standard forms of measurement.

The Rhode Island Early Learning Standards also state that students entering kindergarten may begin to ”use numbers to count as a means for solving problems, predicting, and measuring quantity.” They may be able to use 1–1 correspondence in counting objects and match groups of objects. They may begin to associate the number of objects with the names and symbols for numbers.

Current Learning

Students in kindergarten order sets of objects according to the numbers of objects in each set. They compare and describe two objects according to common attributes. Students compare two groups of objects to determine whether one group has more, less, or the same number of objects as the second group. They compare two written numerals between 1–5 without matching number sets to determine which numerals represent the higher or lower number.

Instruction of this unit is mostly at the developmental level. Sorting categories by count will be taught at the developmental level, while classifying objects into given categories and counting the number of objects in each category is taught at the reinforcement level.

Routines established for oral counting and counting objects with one-to-one correspondence are expected to continue.

Future Learning

Students in grade 1 will organize, represent, and interpret data with up to three categories. In addition to how many objects are in each category, they will determine how many more or less are in one category than another. They will compare and order the lengths of two objects by using a third object.

Students in grade 1 will move from comparing numbers to adding and subtracting in comparing situations, i.e., finding out "how many more" or " how many less" and not just "which is more" or "which is less"(*Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011 p. 5).

Additional Findings

According to the *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), "students in kindergarten classify objects into categories, initially specified by the teacher and perhaps eventually elicited from students. For example, in a science context, the teacher might ask students in the class to sort pictures of various organisms into two piles: organisms with wings and those without wings. Students can then count the number of specimens in each pile. Students can use these category counts and their understanding of cardinality to say whether there are more specimens with things or without wings" (p.5).

Principles and Standards for School Mathematics states that "children in preK-2 begin by comparing and ordering objects using language such as *longer and shorter"* (p. 44). Also, "children should begin to develop an understanding of attributes by looking at, touching, or directly comparing objects. They can compare shoes, placing them side by side, to check which is longer. Adults should help young children recognize attributes through conversation" (p. 103).

Progressions also states, "students in kindergarten classify objects in to categories, initially specified by the teacher and perhaps eventually elicited from students. For example, in a science context, the teacher might ask students in the class to sort pictures of various organisms into two piles: organisms with wings and those without wings. Students can then count the number of specimens in each pile. Students can use these category counts and their understanding of cardinality to ay whether there are more specimens with things or without wings" (p. 5).

"… the standards about comparing numbers focus on students identifying which of two groups has more than (or fewer than, or the same amount as) the other. Students first learn to match the objects in the two groups to see if there are any extra and then to count the objects in each group s and use their knowledge of the count sequence to decide which number is greater than the other (the number farther along in the count sequence). Students learn that even if one group looks as if it has more objects (e.g., has some extra sticking out), matching or counting may reveal a different result" (p. 5).

Kindergarten Mathematics, Quarter 2, Unit 2.3

Composing/Decomposing to 10

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Determine how to separate a number between 1–10 into pairs in more than one way, using objects or drawings.
- Determine the number that needs to be added to another number between 1–9 to make 10, using objects or drawings.

Mathematical practices to be integrated

Attend to precision.

- Calculate accurately and efficiently.
- Express numerical answers with a degree of precision appropriate for the problem context.

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- Analyze relationships mathematically to draw conclusions.

Essential questions

- What are different ways you can separate (a said number between 1–10) into pairs of smaller numbers?
- How can you find out what number needs to be added to _____to make (said number)?
- How can you use objects or pictures to help you solve this problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings~~, and record the answer with a drawing or equation~~.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings~~, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1)~~.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

According to the Rhode Island Early Learning Standards (www.ride.ri.gov/els/resources.asp), students entering kindergarten may begin to "use numbers and counting as a means for solving problems, predicting, and measuring quantity." They may be able to "use one-to-one correspondence in counting objects and matching groups of objects, and begin to associate a number of objects with names and symbols for numbers."

Current Learning

Kindergarten students match the numbers 0–10 to a set that represents that number using one-to-one correspondence. They learn to take a set of objects from 1–10 and separate it into two smaller pairs in multiple ways and to add a certain number of objects to a given set to make a number up to 10. Later in kindergarten, students are expected to record their findings from a composing/decomposing situation using drawings and equations.

This unit is taught at the developmental level.

Routines established for oral counting and counting objects with one-to-one correspondence are expected to continue for this unit.

Future Learning

In grade 1, students will use addition and subtraction of up to three whole numbers within 20 to solve problems involving adding to, taking from, putting together, and taking apart. They will use objects, drawings, and equations with a symbol for the unknown number (such as a question mark). In grade 2, students will solve one- and two-step word problems using addition and subtraction within 100.

Additional Findings

According to the *Progressions for the Common Core State Standards in Mathematics* (K–5 Operations and Algebraic Thinking, May 2011), "in put together /take apart situations, two quantities jointly compose a third quantity (the total), or a quantity can be decomposed into two quantities (the addends). This composition /decomposition may be physical or conceptual. These situations are acted out with objects initially and later children move to conceptual mental actions of shifting between seeing the addends and seeing the total (e.g., seeing children or seeing boys and girls, or seeing red and green apples or all the apples)" (p. 10).

Progressions also states that "put together/take apart situations with both addends unknown play an important role in kindergarten because they allow students to explore various compositions that make each number. This will help students to build the Level 2 embedded number representations used to solve more advanced problem subtypes. Students can find patterns in all of the decompositions of a given number and eventually summarize these patterns for several numbers" (p. 10).

Kindergarten Mathematics, Quarter 2, Unit 2.4

Adding Whole Numbers up to 10

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Show addition using real-world situations.
- Add within 10, using strategies such as objects, drawings, sounds, or words to represent the problem.
- Explore and solve addition word problems with whole numbers from 0–10.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- Use concrete objects or pictures to help conceptualize and solve a problem.
- Check answers to problems using a different method, and continually ask, "Does this make sense?"

Model with mathematics.

- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace (i.e., writing an addition equation to describe a situation).
- Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Essential questions

- What strategy can you use to solve this problem?
- How can you prove your answer?
- How can you use your fingers, objects, drawings, sounds (e.g., claps) to help you find the answer?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition ~~and subtraction~~ with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, ~~or equations~~.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition ~~and subtraction~~ word problems, and add ~~and subtract~~ within 10, e.g., by using objects or drawings to represent the problem.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

According to the Rhode Island Early Learning Standards (www.ride.ri.gov/els/resources.asp), students entering kindergarten may "begin to use numbers and counting as a means for solving problems and they may use words such as more than, less than and add/subtract to express some number concepts."

According to *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), "students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting" (p. 8).

Current Learning

Students investigate addition of whole numbers up to 10 by solving problems that involve joining actions. They use real-life situations to show addition and they use strategies such as drawings, objects, and fingers to represent problems. In quarter 3, students apply skills learned in this unit to subtraction. In quarter 4, they incorporate equations into their work with addition and subtraction.

This unit is taught at a developmental level.

Routines established for oral counting, counting objects with one-to-one correspondence, and composing/decomposing numbers are expected to continue for this unit.

Future Learning

In grade 1, students will work with addition and subtraction problems through number 20 using up to three whole numbers. They will relate counting to addition and subtraction (e.g., counting on 2 to add 2). Students will determine the unknown number in an addition or subtraction equation. They will incorporate place value into addition problems by adding two-digit and one-digit numbers, and adding a two-digit number and a multiple of 10, using models, drawings, or other strategies.

Additional Findings

Principles and Standards for School Mathematics states that "an understanding of addition and subtraction can be generated when young students solve 'joining' and 'take away' problems by directly modeling the situation or by using counting strategies, such as counting on or counting back. Students develop further understandings of addition when they solve missing addend problems that arise from stories or real situations" (p. 83).

According to *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), "students act out adding and subtracting situations by representing quantities in the situation with objects, their fingers, and math drawings. To do this, students must mathematize a real-world situation, focusing on the quantities and their relationships rather than non-mathematical aspects of the situation. Situations can be acted out and/or presented with pictures or words. Math drawings facilitate reflection and discussion because they remain after the problem is solved. These concrete methods that show all of the objects are called Level 1 methods" (p. 8). *Progressions* also states that "students learn and use mathematical and non-mathematical language, especially when they make up problems and explain their representation and solution" (p. 8).