Precalculus, Quarter 1, Unit 1.1
Polynomial Functions

Overview

Number of instructional days: 13  (1 day = 45 minutes)

Content to be learned

- Analyze, interpret, and model the characteristics of polynomial functions and relations through graphs, tables, and equations.
- Determine domain, range; intercepts; increasing and decreasing intervals; rates of change; periodicity; end behavior; maximum and minimum values; continuity; even, odd, or neither function; critical points; and inflection points, both graphically and analytically.
- Calculate operations on functions, including composition and computing inverses algebraically.

Mathematical practices to be integrated

- Use appropriate tools strategically.
- Use technology including, but not limited to, graphing calculators to solve-real world problems involving polynomial functions.
- Use technology to visualize results from problems involving polynomial functions.
- Look for and make use of structure.
- Identify patterns within polynomial functions.
- Make sense of problems and persevere in solving them.
- Work between different representations.

Essential questions

- What are the characteristics of a polynomial equation and its graph?
- What real-world applications of polynomials exist?
- What are the differences between odd and even functions expressed algebraically and graphically?
- How does composing two continuous functions affect the continuity of the new function?
- How are factors, roots, zeros, and solutions of polynomials related?
- How can polynomial equations be described using multiple representations?
- How are a function and its inverse function related?
- What similarities exist between the codomain and the range of a function?
- What differences exist between the codomain and range of a function? How do these differences determine the injectivity/subjectivity of a function?
Grade-Span Expectations

M(F&A)—AM-2 Demonstrates conceptual understanding of linear and nonlinear functions and relations from a set-theoretic perspective, and operations on functions including composition and inverse including computing inverses algebraically; analyzing characteristics of classes of functions and inverse functions (exponential, logarithmic, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes; analyzing properties of functions including injectivity (1-1), surjectivity (onto), critical points and inflection points. Determine graphically and analytically whether a function is even, odd or neither; analyzing informally the idea of continuity and limits; recognizing properties of families of functions including logarithmic and trigonometric, and graphs them; analyzing domain restriction and the effects of it on the function and its properties. (Local)

Clarifying the Standards

Prior Learning

In grade 4, students learned how to write and evaluate simple linear algebraic expressions, and they were introduced to the concept of rate of change. Fifth-grade students wrote linear algebraic expressions involving any two of the four operations, and they solved one-step equations. Students constructed and interpreted graphs in grade 6 in addition to describing the slope of linear relationships.

In grade 7, students were introduced to nonlinear expressions and equations; they solved multistep linear equations involving whole numbers, and they distinguished between constant and varying rates of change using tables or graphs. In grade 8, the concepts of constant and varying rates of change were explored in equations and problem situations. Eighth-graders were also introduced to equations with integers.

Students in grades 9 and 10 were introduced to domain, range, maximum and minimum values, increasing and decreasing intervals by analyzing graphs, tables, and equations. They also simplified polynomial or rational expressions involving exponents, square roots, or absolute value. Solution sets were expressed symbolically and graphically. Transformations of families of functions including exponential and logarithmic functions were analyzed during grade 11.

Current Learning

Students continue to develop, reinforce, and practice linear and nonlinear functions and relations. They analyze properties of functions, including injectivity (1-1); surjectivity (onto); even, odd, or neither; critical points; and inflection points, while analyzing (informally) the concept of continuity and limits.

Future Learning

In calculus, the derivative concept will be applied to the family of polynomial functions. A more rigorous approach of the end behavior of these functions will be analyzed in a variety of representations. Marginal growth and average value of a function in a variety of applications will be explored.

Additional Research Findings

Principles and Standards for School Mathematics speaks to the importance of representing and analyzing mathematical situations and structures. Research indicates that fluency in algebraic concepts lends improves problem-solving abilities in many other areas (p. 300).
Overview

Number of instructional days: 11 (1 day = 45–50 minutes)

Content to be learned

- Analyze, interpret, and model the characteristics and behavior of exponential and logarithmic functions graphically, algebraically, numerically, and analytically.
- Determine domain; range; intercepts; increasing and decreasing intervals; rates of change; periodicity; end behavior; maximum and minimum values; limiting behavior and continuity; even, odd, or neither function; critical points; inflection points; asymptotes; injectivity; surjectivity; and bijectivity, both graphically and analytically.
- Solve equations involving exponential and logarithmic expressions to include converting between logarithmic and exponential form, simplifying logarithmic expressions, expanding and condensing logarithmic and exponential forms.
- Find the general exponential form \( y = ab^x \) and the logarithmic form \( y = a \log_b x \).
- Recognize properties of the families of functions for the exponential and logarithmic functions, to include its domain restriction and its effects both graphically and analytically.
- Determine operations on exponential and logarithmic functions, including finding the inverse algebraically and computing compositions of the functions.
- Apply the Intermediate Value Theorem (IVT) to find exact or approximate solutions of equations or zeros of continuous functions.
- Solve real-world applications of logarithmic and exponential functions to include interest, half-life, and exponential growth/decay.

Mathematical processes to be integrated

- Make sense of problems and persevere in solving them.
- Make conjectures about the form and meaning of the solution and plan a solution pathway for a problem involving logarithmic or exponential functions.
- Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- Work between different representations.
- Use appropriate tools strategically.
- Select and use appropriate technology to solve problems involving exponential or logarithmic functions.
Essential questions

- What are the similarities and differences between the exponential and logarithmic functions?
- What real-world situations can be modeled by exponential and/or logarithmic functions?
- In the exponential and logarithmic functions, \( y = ab^x \) and \( y = a \log_b x \), what effect does changing the \( a \) and \( b \) values have on the graph, if any?

Written Curriculum

Grade-Span Expectations

M(F&A)—AM-2 Demonstrates conceptual understanding of linear and nonlinear functions and relations from a set-theoretic perspective, and operations on functions including composition and inverse including computing inverses algebraically; analyzing characteristics of classes of functions and inverse functions (exponential, logarithmic, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes; analyzing properties of functions including injectivity (1-1), surjectivity (onto), critical points and inflection points. Determine graphically and analytically whether a function is even, odd or neither; analyzing informally the idea of continuity and limits; recognizing properties of families of functions including logarithmic, and graphs them; analyzing domain restriction and the effects of it on the function and its properties. (Local)

Clarifying the Standards

Prior Learning

In grades 4 and 5, students identified, described, or compared situations representing constant rates of change. Slope was introduced in grade 6, and students learned about the concept of dependent and independent variables. In grade 7, students solved problems involving slope and rate of change and informally determined the slope of line from a table or graph. Students also distinguished between constant and varying rates of change.

In grade 8, students determined slopes and intercepts represented in graphs, tables, or problem situations. Students also distinguished between linear relationships and nonlinear relationships. In grades 9-10, students developed conceptual understanding of linear and nonlinear relations. They analyzed characteristics and behavior of functions and relations including intercepts, domain, range, maximum and minimum values, and increasing and decreasing rates. A graphical, numerical (tables, calculator), algebraic, analytical (function behavior), and verbal (writing and notation) approach was further developed. In grade 11, students interpreted, modeled, and solved problems of families of functions.

Current Learning

From a set-theoretic perspective, students further analyze characteristics of exponential and logarithmic functions in depth. These characteristics include domain/range; intercepts; increasing and decreasing intervals; rates of change; periodicity; end behavior; maximum and minimum values; continuity; asymptotes; even, odd or neither functions; critical points; and inflection points, both graphically and
analytically. An informal analysis of continuity and limits of functions in introduced. Domain restrictions on the functions and the effects of them are also incorporated into the lesson. Students study condensing, expanding, converting between, and solving exponential and logarithmic functions and their properties.

**Future Learning**

In calculus, the derivative concept will be applied to exponential and logarithmic functions. A more rigorous approach to the end and asymptotic behavior of these functions will be analyzed in a variety of representations. Exponential growth and decay and logistic applications will be investigated via differential equations. Marginal growth and average value of functions in a variety of applications will be explored.

**Additional Research Findings**

*Science for All Americans* discusses models that can be used to relate to equations (pp. 168–172).


*Beyond Numeracy* compares and contrasts linear and exponential growth and gives real life examples. In addition, models are discussed that can be used to relate to equations (pp. 62–63; 71–72).

### Notes About Resources and Materials
Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Analyze, interpret, and model the characteristics and behavior of rational and radical functions graphically, algebraically, numerically, and analytically.
- Determine domain; range; intercepts; increasing and decreasing intervals; rates of change; periodicity; end behavior; maximum and minimum values; limiting behavior and continuity; even, odd, or neither function; critical points; inflection points; asymptotes; injectivity; surjectivity; and bijectivity, both graphically and analytically.
- Solve equations involving rational and radical expressions.
- Determine operations on exponential and logarithmic functions, including finding the inverse algebraically and computing compositions of the functions.

Mathematical practices to be integrated

- Make sense of problems and persevere in solving them.
- Make conjectures about the form and meaning of the solution and plan a solution pathway for problems involving rational or radical functions.
- Use different representations to analyze and solve problems involving rational and radical functions.

Essential questions

- What are the characteristics of rational and radical functions?
- What are some real-world applications of rational/radical functions?
- How are critical values used to describe a complete graph of a rational function?
- How can rational and radical functions be described using multiple representations?
- How are a rational function and its inverse function related?
- What determines discontinuity in a rational or radical graph?
Grade-Span Expectations

M(F&A)—AM-2 Demonstrates conceptual understanding of linear and nonlinear functions and relations from a set-theoretic perspective, and operations on functions including composition and inverse including computing inverses algebraically; analyzing characteristics of classes of functions and inverse functions (exponential, logarithmic, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes; analyzing properties of functions including injectivity (1-1), surjectivity (onto), critical points and inflection points. Determine graphically and analytically whether a function is even, odd or neither; analyzing informally the idea of continuity and limits; recognizing properties of families of functions including logarithmic and trigonometric, and graphs them; analyzing domain restriction and the effects of it on the function and its properties. (Local)

Clarifying the Standards

Prior Learning

In grade 4, students learned how to write and evaluate simple linear algebraic expressions and were introduced to the concept of rate of change. During grade 5, students wrote linear algebraic expressions involving any two of the four operations and solved one-step equations. Students constructed and interpreted graphs and described the slope of linear relationships in grade 6.

Nonlinear expressions and equations were introduced in grade 7, as well as distinguishing between constant and varying rates of change using tables or graphs. Multistep linear equations were solved involving whole numbers.

In grade 8, the concepts of constant and varying rates of change were explored through equations and problem situations. Students were also introduced to equations with integers, rational expressions, and square roots.

Grade 9 and 10 students were introduced to domain, range, maximum and minimum values, and increasing and decreasing intervals through analyzing graphs, tables, and equations. They also simplified polynomial or rational expressions involving exponents, square roots, or absolute value. Solution sets were expressed symbolically and graphically. Transformations of families of functions, including exponential and logarithmic functions, were analyzed during grade 11. Students analyzed characteristics of functions (polynomial, rational, radical, and exponential), and they practiced adding, subtracting, multiplying, and dividing these functions.

Current Learning

Students were previously exposed to these concepts in grade 11. In grade 12, students extend function analysis and explore properties of rational and radical functions in more detail. A rational function can be viewed as a quotient of polynomial functions with appropriate domain restriction in the denominator. A radical function can be displayed as an nth root of a power function. Students continue to develop, reinforce, and practice rational and radical functions and relations. They analyze properties of these functions, including asymptotes; injectivity (1-1); surjectivity (onto); even, odd, or neither; critical points; and inflection points, while analyzing informally the concept of continuity and limits.
**Future Learning**

In calculus, the derivative concept will be applied to the families of rational and radical functions. From a definition of limit, a more rigorous approach of end and asymptotic behavior of these functions will be analyzed in a variety of representations. Restrictions on domain and range of transformations of these functions, as they apply to daily life, will be addressed and explored. Differential equations, marginal growth, and average or net accumulations of these functions will be investigated using the integral.

**Additional Research Findings**

*NCTM Principles and Standards for School Mathematics* discusses how to represent and analyze mathematical situations and structures using algebraic symbols. According to this source, fluency with algebraic symbolism helps students represent and solve problems in the curriculum. Students need to be fluent in generating algebraic expressions, combining them and re-expressing them in alternative forms (p. 300).

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**Notes About Resources and Materials**
Precalculus, Quarter 1, Unit 1.4
Trigonometry—Special Right Triangles

Overview

Number of instructional days: 6 (1 day = 45–50 minutes)

Content to be learned

- Find missing sides of special right triangles. (45°-45°-90° and 30°-60°-90°)
- Use the trigonometric ratios sine, cosine, and tangent to find missing sides lengths and angle measures in right triangles.
- Solve real-world problems using right triangle trigonometry.

Mathematical practices to be integrated

Model with mathematics.
- Relate learning in mathematics to everyday life situations involving special right triangles.

Attend to precision.
- Use labels and units of measure correctly when working with special right triangle situations.
- Calculate and compute accurately when solving special right triangle problems.

Look for and express regularity in repeated reasoning.
- Look for patterns in problems involving special right triangles to find generalizations.

Essential questions

- What real-life situations can be modeled using right triangle trigonometry?
- What careers and specialized fields use right triangle trigonometry?
- How can right triangles be used to solve problems for other types of triangles?
- What is the connection between similarity of right triangles, trigonometric ratios, and the Pythagorean theorem?
Grade-Span Expectations

M(G&M)—12–5 Applies the concepts of similarity of right triangles with the trigonometric functions defined as ratios of sides of triangles, and uses the ratios of the sides of special right triangles (30°-60°-90° and 45°-45°-90°) to determine the sine, cosine and tangent (30°, 45°, 60°) and solve related problems. (Local)

Clarifying the Standards

Prior Learning

In grade 3, students identified similar shapes. In grade 4, students applied similarity to maps and were introduced to problem solving of similar figures. In grade 5, students described the proportional effect on linear dimensions of triangles and rectangles when scaling up or down, while preserving angle measures. In grade 8, the lengths of the sides of similar triangles were calculated. In geometry, students learned the trigonometric ratios (sine, cosine, tangent) for right triangles.

Current Learning

Students continue to work on concepts of sine, cosine, and tangent and are introduced to 30°-60°-90° and 45°-45°-90° right triangles. Real-world problems are solved using these concepts.

Future Learning

Right triangle trigonometry will be used in surveying, construction, architecture, navigation, and mechanical engineering. An understanding of right triangles is essential to other trigonometry concepts such as non-right triangle trigonometry and circle trigonometry.

Additional Research Findings

Beyond Numeracy states: “The basic insight of elementary trigonometry is that similar triangles, triangles having proportional sides, are such that if you determine the ratio of one side to another in one of these triangles, you will find it be equal to the ratio of the corresponding sides in any of the other similarly shaped ones” (pp. 251–256).