## Algebra 2, Quarter 1, Unit 1.1
### Displays of Data

#### Overview

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<tr>
<th>Number of instructional days:</th>
<th>6 (1 day for assessment)</th>
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<td>(1 day = 45 minutes)</td>
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**Content to be learned**

- Create box and whisker plots, frequency charts, and histograms. (1 day)
- Analyze information about line graphs, bar graphs, scatter plots, circle graphs, box and whisker plots, histograms, and frequency charts (no need to create). Include what information can and cannot be determined from the graph. (1 day)
- Demonstrate knowledge of choosing the correct method to represent data. (1 day)
- Uses technology to analyze data through linear, quadratic and exponential regression to make predictions and justify conclusions. (2 days)

**Mathematical practices to be integrated**

Make sense of problems and persevere in solving them.

- Analyze givens, constraints, relationships, and goals to determine the best ways to represent data.
- Graph data and search for regularity or trends.

Use appropriate tools strategically.

- Use statistical software to analyze data through regression.

**Essential questions**

- Why is data important in making predictions?
- In what kind of problem situation would linear models be used?
- What information can be determined from box-and-whisker plots and histograms, and what information cannot be determined from those representations?
- What are the advantages or disadvantages of choosing one data representation over the other?
- In what real-world situation would a linear model be used?
Grade-Span Expectations

M(DSP)–12–3 Organizes and displays one- and two-variable data using a variety of representations (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts, linear, quadratic, and exponential regression functions) to analyze the data to formulate or justify conclusions, make predictions, or to solve problems with or without using technology. (Local)

Clarifying the Standards

Prior Learning

In grades 3–4, students organized and displayed data using bar graphs to answer questions, make predictions, and solve problems. In grades 5–6, line graphs were introduced and students determined the best method of representing data. In grades 7–8, scatter plots and circle graphs were introduced and used to draw conclusions and make predictions. In grades 8–9, the process of selecting the best method of representation was reinforced. Also in grades 7–10, histograms and frequency charts were introduced and interpreted only.

Current Learning

Students create histograms and frequency charts for the first time and continue to make conclusions and predictions. Students also are introduced to using linear, quadratic, and exponential regression from displays of data to draw conclusions and make predictions; these concepts are reinforced and mastered at this level. Students learn to organize and display data with and without technology.

Future Learning

Students will use these skills in precalculus where they will expand their regression knowledge. Students will learn how these skills can be used in the business world, for example, to improve productivity, to improve overall company improvement, and to examine trend analysis. There skills also have science and general research applications. Students will expand general knowledge by reading newspapers, magazines, etc. The content learned in this lesson can also be an introduction to higher-level statistics and business classes.

Additional Research Findings

Principles and Standards for School Mathematics includes relevant information on how students gain a deeper understanding of variables and how those variables change conclusions. It also speaks to formulating questions that can be addressed with relevant data (pp. 324–331).

There is further discussion about describing, interpreting, organizing, and displaying data in A Research Companion to Principles and Standards for School Mathematics (pp. 193–215).
Algebra 2, Quarter 1, Unit 1.2

Real Numbers

Overview

Number of instructional days: 5 (1 day for assessment) (1 day = 45 minutes)

Content to be learned

• Master conceptual understanding of the real number system and its subsets. (1 day)
• Demonstrate knowledge of comparing and ordering. (1 day)
• Demonstrate conceptual understanding of the closure property over arithmetic operations. (1 day)
• Demonstrates an understanding of why a real number is rational by using decimal expansion. (1 day)

Mathematical practices to be integrated

Reason abstractly and quantitatively.
• Make sense of quantities and their relationships in problem situations.

Look for and make use of structure.
• Look closely to discern a pattern or structure in the real number system.
• Use the decimal expansion of a real number to understand its rationality.

Essential questions

• What are the similarities and differences between rational and irrational numbers?
• What does it mean for a set to be closed?
• When is it appropriate to use estimates of numbers, versus an exact value, in a real-life situation?
Grade-Span Expectations

M(N&O)–12–1 **Demonstrates conceptual understanding of rational numbers** by knowing why a real number is rational if and only if the number’s decimal expansion eventually repeats or terminates. (Local)

M(N&O)–12–2 **Demonstrates understanding of the relative magnitude of real numbers** by solving problems that involve ordering or comparing any subset of the real numbers. (Local)

M(N&O)–12–7 **Makes appropriate estimates** in a given situation by determining the level of accuracy needed and analyzing the accuracy of results. (Local)

   (IMPORTANT: *The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.*)

M(N&O)–12–8 **Applies properties to** determine whether a given subset of numbers is closed under a given arithmetic operation. (Local)

M(N&O)–AM-1 **Demonstrates conceptual understanding of the real number system** as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and as points on a number line. Determines whether the decimal expansion of a rational number given in fractional form eventually repeats or terminates (without using a calculator). (Local)

Clarifying the Standards

*Prior Learning*

Students began with conceptual understanding of rational numbers in kindergarten and continued building on this knowledge every year. From kindergarten through grade 2, they compared and ordered whole numbers. In grade 3, they added positive fractions. In grade 4, decimals were added. In fifth grade, percents were added to the comparing and ordering of whole numbers and fractions.

Exponents, rational numbers, “equal to,” and inequalities were introduced in sixth grade. Absolute value and scientific notation were added in grade 7. In grade 8, students were introduced to comparing the basic irrational numbers and square roots.

Students began making estimates of objects in kindergarten through grade 2. In grades 3–5, they identified when estimation was appropriate and learned how to select appropriate method of estimation. In grades 5–10, they determined the levels of accuracy needed. Students in grades 9–10 applied properties, and compared, contrasted, and simplified to solve problems.

Students applied commutative and identity properties of addition in grade 1. Grade 2 added the associative property. Grade 3 and 4 introduced the multiplication properties. Grade 5 included study of the distributive property and divisibility. In grade 6, prime factorization, the multiplicative identity and the additive inverse are included. In grade 7, students learned multiplicative inverse. In grade 8, nontraditional properties were added.
Current Learning

In grade 11, students explore how to determine whether a real number is rational by decimal expansion (whether the number repeats or terminates). Students also develop and master closure.

Future Learning

In grade 12, infinite decimal expansion will be explored as an extension of rational numbers representing real numbers, and properties of numbers will be applied to matrices.

Additional Research Findings

According to Principles and Standards for School Mathematics, “students need to understand the numbers to explore new systems” (vectors, matrices)” (pp. 290–293).

Additionally, Science for All Americans emphasizes the importance of students at this level being able to compare different number systems and operations on numbers (pp. 130–132).
Algebra 2, Quarter 1, Unit 1.3
Operations and Simplifying Roots
(square, nth)

Overview

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Content to be learned

- Simplify expressions involving radicals. (1 day)
- Add and subtract expressions involving radicals. (1 day)
- Multiply expressions involving radicals by using both distributing and the foil method. (1 day)
- Divide radical expressions, including rationalizing denominators (including conjugates). (2 days)

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Monitor and evaluate progress when solving a problem involving radicals and change course if necessary.
- Transform expressions involving radicals to get the information needed.

Essential questions

- How are radicals related to natural number exponents? (e.g., square root, cubic root, fourth root, etc.)
- How is adding or subtracting expressions involving radicals the same or different from multiplying expressions involving radicals?
- Why is it important to use a conjugate when dividing radical expressions?
- How is finding the cube root of a number different from finding the square root?
Written Curriculum

Grade-Span Expectations

M(F&A)–12–3 Demonstrates conceptual understanding of algebraic expressions by manipulating, evaluating, and simplifying algebraic and numerical expressions; adding, subtracting, multiplying and dividing polynomials; adding, subtracting, multiplying and dividing rational expressions; simplifying complex fractions; factoring quadratic and higher degree polynomials, including difference of squares; applying properties of logarithms (e.g. \( \log_a b^n = n \log_a b \), \( a^{\log_a b} = b \)) and converting between logarithmic and exponential forms; manipulating, evaluating, and simplifying expressions involving rational exponents and exponential forms; simplifying complex fractions; factoring quadratic and higher degree polynomials, including difference of squares; applying properties of logarithms (e.g. \( \log_a b^n = n \log_a b \), \( a^{\log_a b} = b \)) and converting between logarithmic and exponential forms; manipulating, evaluating, and simplifying expressions involving rational exponents and radicals and converting between expressions with rational exponents and expressions with radicals. (Local)

Clarifying the Standards

Prior Learning

Students were introduced to simplifying perfect squares in eighth grade. In grades 9–10, students reinforced their skills in simplifying square roots including an introduction to non-perfect squares. The operations using square roots were developed.

Current Learning

Students master operations involving square roots. They are introduced to and master the operations and simplification of nth roots.

Future Learning

Students will apply their knowledge of radicals in precalculus and calculus classes. Applications of roots occur in physics, finance, conics, and other various science applications. In mathematics, there are specific formulas—such as the Pythagorean theorem, length measurements (distance formula), area, volume, and the quadratic formula—that require an understanding of radicals.

Additional Research Findings

According to Principles and Standards for School Mathematics, students need to understand the numbers to explore new systems (vectors, matrices) (pp. 290–294). And Beyond Numeracy by John Allen Paulos, states the importance of emphasizing applications in geometry that are solved using which will use radicals (volume, area, the Pythagorean theorem, and the quadratic formula) (pp. 18–23, 192–194, 198–201).
Algebra 2, Quarter 1, Unit 1.4
Polynomial Functions

Overview

**Number of instructional days:** 7 (1 day for assessment)  
(1 day = 45 minutes)

**Content to be learned**

- Analyze polynomial functions by examining maximum, minimum, intercepts, increasing, and decreasing (including non-polynomial functions). (1 day)
- State the domain and range of polynomial and non-polynomial functions using interval notation. (2 days)
- Classify polynomial functions by degree and number of terms, identifying the leading coefficient, degree, and end behavior. (1 day)
- Analyze polynomial functions algebraically and graphically, including using technology, and determine the degree of a polynomial function from a table using differences. (2 days)

**Mathematical practices to be integrated**

- Use appropriate tools strategically.
- Analyze graphs of polynomial functions generated using a graphing calculator.
- Use technology to determine characteristics of polynomial functions.
- Attend to precision.
- Communicate precisely to others regarding problems involving polynomial functions.
- Use clear definitions in discussion with others and in reasoning regarding problems involving polynomial functions.

**Essential questions**

- What is the connection between the graphical and algebraic representation of polynomial functions?
- What are the similarities and differences between polynomial functions with varying degrees?
- How are zeros, roots and x-intercepts of polynomial functions related?
- How do the domain and range of polynomial functions compare to non-polynomial functions?
Written Curriculum

Grade-Span Expectations

M(F&A)–12-2 Demonstrates conceptual understanding of linear and nonlinear functions and relations by representing and analyzing functions in several ways; recognizing properties of functions and characteristics properties of families of functions; applying knowledge of functions to interpret, model, and solve problems; analyzing characteristics of classes of functions (polynomial, rational, and exponential) to include domain, range, intercepts, increasing and decreasing intervals and rates of change; representing functions numerically, algebraically, graphically, and verbally (i.e. in written words), recognizing properties of a function from these representations, and transfers information from one representation to another; graphing polynomial, rational and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes; applying knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in natural and social sciences. (Local)

Clarifying the Standards

Prior Learning

In grade 4, students demonstrated conceptual understanding of linear relationships and constant rates of change. This continues through sixth grade, where the change of the independent variable affects the dependent variable was introduced. Students in grades 7–8 were introduced to nonlinear relationships and varying rates of change, and there students learned the differences between constant and varying rates of change. Students in ninth grade were required to understand and apply linear and nonlinear relationships, including domain, range, maximum and minimum, intercepts, increasing, and decreasing.

Current Learning

Students master knowledge of the characteristics of linear and nonlinear functions, including domain, range, maximum, minimum, intercepts, increasing, and decreasing.

Future Learning

Students will demonstrate conceptual understanding of one-to-one and onto, including critical points, inflection points, end behavior, odd, even, continuity, and limits. They will also analyze domain restrictions. These topics will expand to logarithmic and trigonometric functions.

Additional Research Findings

Benchmarks for Science Literacy, in the chapter titled The Mathematical World, explains the science relationship and use of polynomial functions. This source includes examples, such as rate of change (pp. 218–222).
Algebra 2, Quarter 1, Unit 1.5
Rational Functions

Overview

Number of instructional days: 6 (1 day for assessment)
(1 day = 45 minutes)

Content to be learned

- Find the domain, range, and vertical asymptotes, and horizontal asymptotes of a rational function by analyzing the graph and the function in factored form. (2 days)
- Mark the increasing and decreasing intervals and intercepts on the graph of a rational function. (1 day)
- Create graphs of rational functions that are in factored form. (2 days)

Mathematical practices to be integrated

Model with mathematics.
- Model problem situations using rational functions.

Use appropriate tools strategically.
- Use a graphing calculator or computer software to graph and analyze rational functions.

Essential questions

- How do rational functions differ from polynomial functions?
- How do you know there are discontinuities in a rational function either by looking at the graph or analyzing the equation?
Grade-Span Expectations

M(F&A)–12-2 **Demonstrates conceptual understanding of linear and nonlinear functions and relations** by representing and analyzing functions in several ways; recognizing properties of functions and characteristics properties of families of functions; applying knowledge of functions to interpret, model, and solve problems; analyzing characteristics of classes of functions (polynomial, rational, and exponential) to include domain, range, intercepts, increasing and decreasing intervals and rates of change; representing functions numerically, algebraically, graphically, and verbally (i.e. in written words), recognizing properties of a function from these representations, and transfers information from one representation to another; graphing polynomial, rational and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes; applying knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in natural and social sciences. (Local)

Clarifying the Standards

**Prior Learning**

In the grades 4–6, students demonstrated conceptual understanding of linear relationships and constant rates of change. In sixth grade, students learned how the change of the independent variable affects the dependent variable. Students in grades 7–8 were introduced to nonlinear relationships and varying rates of change, and they learned the differences between constant and varying rates of change. In grade 9, students learned and applied linear and nonlinear relationships, including domain, range, maximum, minimum, intercepts, increasing, and decreasing.

**Current Learning**

Students develop and master knowledge of the characteristics of rational functions (domain, range, asymptotes, maximum, minimum, intercepts, increasing, and decreasing).

**Future Learning**

The students will demonstrate conceptual understanding of one-to-one and onto, including critical points, inflection points, end behavior, odd, even, continuity, and limits. They will analyze domain restrictions. These topics will expand to logarithmic and trigonometric functions.

**Additional Research Findings**

*A Research Companion to Principles and Standards for School Mathematics* highlights how the visual attributes of graphs helps students construct meaning (pp. 250–262). And *Principles and Standards for School Mathematics* discusses how students analyze the same function—using technology—in different ways depending how the functions has changed (p. 302).
Exponential Functions

Overview

Number of instructional days: 6 (1 day for Assessment) (1 day = 45 minutes)

Content to be learned

• Distinguish between exponential growth and exponential decay from a graph and from an equation. (1/3 day)
• State the domain and range of an exponential function. (1/3 day)
• Determine the intercepts and the intervals where the graph of an exponential function is increasing and decreasing. (1/3 day)
• Create graphs of exponential functions. (2 days)
• Write equations of exponential functions given a table of values. (The table must include x=0). (2 days)

Mathematical practices to be integrated

Reason abstractly and quantitatively.
  • Decontextualize a given situation involving an exponential function and represent it symbolically.
  • Write a context to fit a given exponential function.
Model with mathematics.
  • Model a real-world situation with an exponential function.

Essential questions

• What are the similarities and differences between polynomial functions and exponential functions?
• What are the similarities and differences between exponential growth and exponential decay?
Written Curriculum

Grade-Span Expectations

M(F&A)–12-2 Demonstrates conceptual understanding of linear and nonlinear functions and relations by representing and analyzing functions in several ways; recognizing properties of functions and characteristics of families of functions; applying knowledge of functions to interpret, model, and solve problems; analyzing characteristics of classes of functions (polynomial, rational, and exponential) to include domain, range, intercepts, increasing and decreasing intervals and rates of change; representing functions numerically, algebraically, graphically, and verbally (i.e. in written words), recognizing properties of a function from these representations, and transfers information from one representation to another; graphing polynomial, rational and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes; applying knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in natural and social sciences. (Local)

Clarifying the Standards

Prior Learning

In grade 8, nonlinear relationships were introduced and represented in tables, graphs, equations, or problem situations. In Algebra I, students demonstrated understanding of the characteristics of multiple types of nonlinear functions including intercepts, domain, and range, and slope. They analyzed the differences between constant and variable rates of change.

Current Learning

In Algebra 2, students continue to work with and master an understanding of properties and characteristics of exponential functions, including domain, range, intercepts, increasing and decreasing intervals, rates of change/slope and logarithms. Students use multiple representations including algebraic, word problems, and tabular.

Future Learning

In precalculus, students will delve deeper in graph analysis of exponential functions and their relationship to logarithmic functions. They also will use exponential functions to solve word problems, and they will study set theory and inverse functions. Real-world connections will be made to financial concepts such as compound interest.

Additional Research Findings

Beyond Numeracy by John Allen Paulos contains useful information on exponential growth (pp. 62-63, 71-72. It also discusses the history of e, the standard definition of e as a limit, and other definitions. Real-life examples of exponential growth and decay, including banking functions, are explored.