Algebra 2, Quarter 3, Unit 3.1
Operations with Complex Numbers

Overview

Number of instructional days: 9 (1 day for review and 1 day for assessment) (1 day = 45 minutes)

Content to be learned

• Simplify the square roots of negative numbers. (2 days)
• Add, subtract, and multiply complex numbers. (2 days)
• Find the conjugate of a complex number. (0.5 day)
• Divide complex numbers, including rationalization of the denominator using the conjugate. (1.5 days)
• Plot complex numbers on a complex plane. (0.5 day)
• Demonstrate the absolute value of a complex number. (0.5 day)

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.
• Think about simpler problems to help solve move complex problems.
• Work between different representations.
• Look for and make use of structure.
• Break down complicated problems into smaller, less complex problems.

Essential questions

• Why do imaginary numbers exist?
• After performing operations with complex numbers, what determines when the expression is in its simplest form?
• How does the division of pure imaginary numbers compare to the division of complex numbers?
• Why is it important to use a conjugate when dividing by a complex number?
• How can rectangular coordinates be used to manipulate complex numbers?
Written Curriculum

Grade Span Expectations

M(N&O)–AM–4 Accurately solves problems and demonstrates understanding of complex numbers by interpreting them geometrically and by computing with them (e.g., adding, multiplying, dividing, finding the nth root, or by finding conjugates). Understands complex numbers as an extension of the real numbers (e.g. arising in solutions of polynomial equations). Manipulates complex numbers using rectangular and polar coordinates. Knows the fundamental theorem of algebra and knows that non-constant polynomials always factor into linear factors over the complex numbers. (Local)

Clarifying the Standards

Prior Learning

Students have solved problems by computation since grade 3. Students worked with real numbers through grade 10.

Current Learning

Students reinforce the concept of nth roots to expand into imaginary numbers. They master addition, subtraction, and multiplication of complex numbers as an extension of the real number system. Students are introduced to finding the conjugate of a complex number and learn to divide complex numbers with and without the conjugate. In units of study 3.2 and 3.3, students find complex zeros.

Future Learning

In grade 12, students will manipulate complex numbers using rectangular and polar coordinates. Also, students will reinforce these concepts in other college courses, computer graphics, fractal geometry, and electrical engineering.

Additional Research Findings

Beyond Numeracy by John Allen Paulos discusses the origins of i and the expansion of the number system to complex numbers. It also shows that the fundamental theorem of algebra works in that solutions may be complex numbers (pp. 116–117).
Algebra 2, Quarter 3, Unit 3.2
Solving Quadratic Equations

Overview

Number of instructional days: 13 (1 day for review and 1 day for assessment)
(1 day = 45 minutes)

Content to be learned
• Factor the sum of two perfect squares. (0.5 day)
• Analyze graphs to determine the different types of solutions of quadratic equations (e.g., two real, double root, imaginary). (0.5 day)
• Solve quadratic equations graphically and from tables using both vertex and standard form \((-b/2a)\). (1 day)
• Solve quadratic equations by factoring, completing the square, and using the quadratic formula. (7 days)
• Solve quadratic equations using a graphing calculator to find the zeros. (1 day)
• Solve a quadratic equation graphically, using technology, or by interpreting the equation as two equal functions and graphing the two functions. (1 day)

Mathematical practices to be integrated
Make sense of problems and persevere in solving them.
• Read, plan, solve, and check; modify and revise, if necessary.
• Check the reasonableness of the solution; check answers by using multiple methods.

Use appropriate tools strategically.
• Use tools and technology to explore and deepen understanding of mathematics and be able to detect errors.
• Understand the limitation of tools and technology.

Essential questions
• What is the relationship between factors, roots, zeros, solutions, and \(x\)-intercepts?
• What is the disadvantage of using the graphing calculator when solving a quadratic equation?
• What is the difference between factoring the sum of two perfect squares and factoring the difference of two perfect squares?
• What are the advantages of learning how to solve quadratic equations by using the quadratic formula, completing the square, and factoring?
• What is the link between the table of values and the graph of a quadratic function (with and without a graphing calculator)?
• What are some real-world examples in which a quadratic model may be used?
Grade Span Expectations

M(F&A)–AM–3 Demonstrates conceptual understanding of algebraic expressions by using the remainder theorem, the factor theorem and rational root theorem for polynomials; by factoring polynomials over integer, rational, real and complex numbers. (Local)

M(F&A)–12–4 Demonstrates conceptual understanding of equality by solving equations and systems of equations or inequalities and interpreting the solutions algebraically and graphically; by factoring, completing the square, using the quadratic formula, and graphing quadratic functions to solve quadratic equations; solving and interpreting solutions of equations involving polynomial, rational, and radical expressions; analyzing the effect of simplifying radical or rational expressions on the solution set of equations involving such expressions. (e.g. \(x^2/2x = x\) for \(x \neq 0\); finding approximate solutions to equations by graphing each side as a function using technology. \([\text{Understand that any equation in } x \text{ can be interpreted as the equation } f(x) = g(x) \text{ and interpret the solutions of the equation as the } x\)-value(s) of the intersection point(s) of the graphs of } y = f(x) \text{ and } y = g(x)\].) solving 2x2 and 3x3 systems of linear equations and graphically interpreting the solutions; solving systems of linear and quadratic inequalities; solving and graphically interpreting solutions systems of equations involving nonlinear expressions. (Local)

Clarifying the Standards

Prior Learning

In grades 6–10, students learned about linear and nonlinear patterns. In grades 9–10, students were introduced to linear and nonlinear functions. In grade 4, students were introduced to the words factor and multiple, and in grades 6–8, students used prime factorization. In grade 7, students learned about perfect squares. Grade 8 students learned about perfect squares and cube roots. In grades 8–10, students simplified expressions with square roots. In grade 9, students were introduced to multiplication of binomials and multiplication of a monomial by a binomial, and students were introduced to quadratics.

Current Learning

In unit of study 2.4, students are taught how to factor quadratic expressions. In this unit of study, students are introduced to and reinforce concepts of solving quadratic equations by graphing, tables, factoring, completing the square, and the quadratic formula, with and without technology. In unit of study 3.3, students solve higher-order polynomials.

Future Learning

In grade 12, students will continue solving polynomials. They will also solve polynomial equations in calculus, business, computers, engineering, and science.

Additional Research Findings

Beyond Numeracy by John Allen Paulos discusses the quadratic formula being the first theorem proved in high school algebra. The book also discusses approximating versus exact roots and links this content to physics problems (pp. 198–199).

Principles and Standards for School Mathematics talks about quadratic functions, their properties, and transformations that occur. There is information about changing the quadratic into vertex form (pp. 299–301).
Algebra 2, Quarter 3, Unit 3.3
Polynomials

Overview

Number of instructional days: 10 (1 day assessment) (1 day = 45 minutes)

Content to be learned

• Add, subtract, and multiply polynomial expressions. (0.5 day)
• Simplify a polynomial expression using long division and synthetic division. (1.5 days)
• Solve polynomial equations using the remainder theorem, rational root theorem, factor theorem, and intermediate value theorem. (3 days)
• Solve polynomial equations algebraically and graphically, and interpret the results. (1 day)
• Use technology to approximate solutions to polynomial equations. (1 day)
• Create polynomial equations given the roots (include complex roots). (2 days)

Mathematical practices to be integrated

Use appropriate tools strategically.
• Use technology to visualize results.
• Use various tools, including technology, to help them solve problems.
• Understand the limitation of tools and technology.

Make sense of problems and persevere in solving them.
• Think about simpler problems to help solve more complex problems.
• Read, plan, solve, and check; modify and revise if necessary.

Essential questions

• Why is it necessary to learn both synthetic division and long division?
• How are adding and subtracting polynomial expressions different from multiplying polynomial expressions?
• What are the disadvantages of using technology to solve polynomial equations?
• How can the remainder theorem, factor theorem, rational root theorem, and intermediate value theorem be used to solve polynomial equations?
• In what real-world situations would polynomials be used?
Written Curriculum

Grade Span Expectations

M(F&A)–AM–3 Demonstrates conceptual understanding of algebraic expressions by using the remainder theorem, the factor theorem and rational root theorem for polynomials; by factoring polynomials over integer, rational, real and complex numbers. (Local)

M(F&A)–12–3 Demonstrates conceptual understanding of algebraic expressions by manipulating, evaluating, and simplifying algebraic and numerical expressions; adding, subtracting, multiplying and dividing polynomials; adding, subtracting, multiplying and dividing rational expressions; simplifying complex fractions; factoring quadratic and higher degree polynomials, including difference of squares; applying properties of logarithms (e.g. \( \log_a b^n = n \log_a b \), \( a^{\log_a b} = b \)) and converting between logarithmic and exponential forms; manipulating, evaluating, and simplifying expressions involving rational exponents and radicals and converting between expressions with rational exponents and expressions with radicals. (Local)

M(F&A)–AM–4 Demonstrates conceptual understanding of equality by solving equations and verifying identities involving trigonometric expressions; solving, graphing and interpreting equations involving exponential and logarithmic expressions; interpreting systems as matrix equations and solving them by computing the appropriate matrix inverse and multiplication, with or without technology; applying the intermediate value theorem to find exact or approximate solutions of equations or zeros of continuous functions. (Local)

M(F&A)–12–4 Demonstrates conceptual understanding of equality by solving equations and systems of equations or inequalities and interpreting the solutions algebraically and graphically; by factoring, completing the square, using the quadratic formula, and graphing quadratic functions to solve quadratic equations; solving and interpreting solutions of equations involving polynomial, rational, and radical expressions; analyzing the effect of simplifying radical or rational expressions on the solution set of equations involving such expressions. (e.g. \( x^2/2x = x \) for \( x \neq 0 \)); finding approximate solutions to equations by graphing each side as a function using technology. [Understand that any equation in \( x \) can be interpreted as the equation \( f(x) = g(x) \) and interpret the solutions of the equation as the \( x \)-value(s) of the intersection point(s) of the graphs of \( y = f(x) \) and \( y = g(x) \)]. solving 2x2 and 3x3 systems of linear equations and graphically interprets the solutions; solving systems of linear and quadratic inequalities; solving and graphically interpreting solutions systems of equations involving nonlinear expressions. (Local)

Clarifying the Standards

Prior Learning

In grade 4, students used variables to represent unknowns, and in grades 5–8, students worked with evaluating expressions. In grade 9, students were introduced to adding, subtracting, and multiplying polynomial expressions.
Current Learning

In unit of study 3.2, students learn to rewrite polynomial equations as two equal functions and to solve by graphing each function and interpreting the intersection of the graphs. Students review and master addition, subtraction, and multiplication of polynomial expressions; division of polynomials by synthetic division and long division is introduced. Students learn how to solve polynomial equations using the remainder theorem, factor theorem, and rational root theorem.

Future Learning

Students will continue to study and master polynomial equations in grade 12. Also, students will use polynomials in business, finance, engineering, and computer science.

Additional Research Findings

A Research Companion to Principles and Standards for School Mathematics discusses functions and solutions on a graph (pp. 132–133).

Principles and Standards for School Mathematics discusses why students learn algebra to expand their repertoire of functions (p. 297).
Algebra 2, Quarter 3, Unit 3.4
Rational Exponents and Applications

Overview

Number of instructional days: 8 (1 day review and 1 day assessment) (1 day = 45 minutes)

Content to be learned

- Evaluate rational exponents by converting between exponential and radical forms, with and without technology. (2 days)
- Simplify expressions with rational exponents. (2 days)
- Use rational exponents to solve compound interest problems. (2 days)

Mathematical practices to be integrated

Attend to precision.
- Use precise mathematical vocabulary, clear and accurate definitions, and symbols to communicate efficiently and effectively.
- Calculate and compute accurately (including technology).

Look and make use of structure.
- Apply prior learning to new situations.
- Identify patterns and structures.

Essential questions

- In addition to compound interest, what are some real-world instances when rational exponents can be used?
- How are rational exponents related to radicals? Be specific.
Written Curriculum

Grade Span Expectations

M(N&O)—12–4 Accurately solves problems involving scientific notation or uses significant digits to assess the precision of an answer. Interprets rational exponents and their relation to radicals; computes by hand in simple cases (e.g. \(4^2\)), and using a calculator when appropriate. Interprets numbers given in scientific notation and carries out computations of them with and without a calculator. Solves problems involving compound interest. (Local)

M(F&A)—12–3 Demonstrates conceptual understanding of algebraic expressions by manipulating, evaluating, and simplifying algebraic and numerical expressions; adding, subtracting, multiplying and dividing polynomials; adding, subtracting, multiplying and dividing rational expressions; simplifying complex fractions; factoring quadratic and higher degree polynomials, including difference of squares; applying properties of logarithms (e.g. \(\log_a b^n = n \log_a b\), \(a^{\log_a b} = b\)) and converting between logarithmic and exponential forms; manipulating, evaluating, and simplifying expressions involving rational exponents and radicals and converting between expressions with rational exponents and expressions with radicals. (Local)

Clarifying the Standards

Prior Learning

In grades 3–5, students demonstrated conceptual understanding of rational numbers. In grades 4–5, they encountered problems using addition and subtraction of positive proper fractions with like denominators. In grade 6, students performed operations on all types of fractions. In grades 7 and 8, they learned about exponents; exponential properties were introduced. Students also determined the square roots of perfect squares. In grades 9 and 10, students simplified problems with rational exponents.

Current Learning

Students are introduced to and master computations with rational exponents. They see the connection between rational exponents and radicals. Students solve problems involving compound interest. Technology is used where appropriate.

Future Learning

Students will continue to use rational exponents in future math courses, as well as business, banking, finance, computer science, and the sciences.

Additional Research Findings

Becky Numeracy by John Allen Paulos gives an example of compound interest and how money can grow over the years (p. 71), and offers a quick overview of the law of exponents using rational exponents (p. 219).